IDENTIFICATION OF THE EQUILIBRIUM EXCHANGE RATE PASS-THROUGH EFFECT IN COINTEGRATED VAR WITH AN APPLICATION TO THE EURO AREA* 

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ABSTRACT: The exchange rate pass-through is of considerable importance for policy makers in open economies. Based on work of Johansen (2002) this paper develops the conditions for the identification of equilibrium pass-through effect in cointegration framework. In addition, I specify the restrictions for testing the perfect equilibrium pass-through. The method is illustrated on the Euro area data and the pass-through effect of the Euro effective exchange rate. The results show that conditional on the type of economic shocks that lead to a permanent change in the exchange rate, the equilibrium pass-through effect can be both very low and high.

Keywords: exchange rate pass-through effect, identification, cointegration analysis

JEL Classification: E42, E52, E58, C32

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1. Introduction

Exchange rate pass-through is defined as the change in prices caused by the change in the nominal exchange rate. The pass-through effect operates broadly through three basic channels: (1) a direct effect through prices of imported goods in the CPI; (2) an effect through prices of imported intermediate goods; and (3) an effect through price setting and expectations that include also the expected responses of monetary policy (Garcia and Restrepo, 2001). Estimation of pass-through effect leads to important policy implications in the choice of exchange rate regimes. At the same time, it represents a significant challenge in empirical work.

The literature has developed two broad approaches to estimation of exchange rate pass-through effect. The first approach is based on single-equation estimation in large cross-country panels. A notable example are Campa and Goldberg (2002) (see also Campa and Goldberg, 2005) who estimate a simple single-equation model for 25 OECD countries over the period 1975 to 1999 and measure the pass-through effect (to import prices) with the coefficient on the nominal exchange rate. Goldfajn and Werlang (2000) study the relationship between exchange rate depreciations and inflation for 71 countries in the period 1980 to 1998. Choudhri and Hakura (2001) extend the study of Goldfajn and Werlang (2000) and try to establish the role of the exchange rate regime in determining the extent of pass-through in 71 countries in the period 1979 to 2000. Single equation approach is used also by Darvas (2001) and Mihaljek and Klau (2008) who study emerging economies.

2 The views expressed in this paper are solely the responsibility of the authors and should not be interpreted as reflecting the views of the Bank of Slovenia.

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VAR pass-through is typically measured by means of impulse responses of prices to an identified exchange rate shock. The problem with such an approach is that it gives only a partial estimate of pass-through. As shown by Corsetti and Dedola (2005) we can observe as many (different) measures of pass-through effect as there are identified structural shocks. Concentrating on price responses only to an exogenous exchange rate shock neglects other sources of stochastic variation in the nominal exchange rate and prices. In addition, exchange rate changes need not occur only as consequences of stochastic shocks, but they can also reflect systematic changes in policy like the change in the inflation target. All such changes are not accounted for in a typical SVAR analysis.

This paper draws on Coricelli et al. (2006) and considers the estimation of pass-through effect with the cointegrated vector autoregression model (CVAR). Such an approach has a number of important advantages. Price series are commonly integrated at least of order one, which calls for an explicit test for cointegration. From an economic point of view, neglecting cointegration is surprising since long-run co-movement of prices and exchange rate is borne out by theory. Neglecting cointegration when it is genuinely present means neglecting the intrinsic meaning of equilibrium long-run relationship between the nominal exchange rate and prices.

CVAR methodology has been used in previous studies (see for example Kim, 1998; Billmeier and Bonato, 2002; Babecka-Kucharcukova, 2009 and Beirne and Bijsterbosch, 2009). These studies, however, do not discuss the issue of interpretation of cointegration coefficients as in Johansen (2002), which implies that the estimates of the pass-through presented in these studies may not be properly identified. Lack of identification means that cointegration coefficient estimates cannot be interpreted as equilibrium coefficients of pass-through.

This paper contributes to the literature by developing the criteria for identification of equilibrium exchange rate pass-through within the cointegration framework. I show that identification depends on cointegration rank. Moreover, the estimated cointegration rank suggest how many other endogenous variables, in addition to prices, are affected by a permanent change in the exchange rate. For instance, the extent of exchange rate pass-through to prices may differ if we allow for accompanying equilibrium changes in output or in interest rates, which is in line with the theoretical analysis of Corsetti and Dedola (2005). In the first case we measure the effect of the exchange rate on prices conditional on a permanent real shocks, while in the second conditional on a financial shock. Finally, I show how the proposed identification framework can be used to test for perfect exchange rate pass-through.

The method is illustrated on the Euro area data. The Euro area case is interesting from the point of view of unconventional monetary policy measures initiated by the European central bank in the fall of 2014 with important expected effects operating through the exchange rate channel. The empirical analysis in the paper shows that pass-through of the Euro effective exchange rate to consumer prices is the highest in case of exchange rate changes accompanied by long-run changes in real output. In such a case, the test cannot reject the hypothesis of a perfect pass-through. Conditional on equilibrium changes in interest rates (financial shock) or foreign prices (nominal shock) the extent of exchange rate pass-through is limited.

The paper proceeds as follows. Section 2 discusses the theoretical conditions for identification of the exchange rate pass-through effect in a cointegrated VAR model. Section 3 presents the empirical results, while Section 4 concludes.

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2A different use of SVAR analysis is found in Choudri, Faruqee and Hakura (2002). Their empirically observed impulse responses of prices to an exchange rate shock are used not to measure pass-through directly but as a benchmark for simulated responses obtained from calibrated theoretical model under different assumptions about nominal rigidities in the economy.
2. Identification of equilibrium exchange rate pass-through effect

This section presents the analysis of the equilibrium exchange rate pass-through effect to (consumer) prices within the cointegration framework. Compared to existing studies of pass-through the analysis introduces two novelties. First, within I(1) cointegration analysis the paper offers a formal discussion of identification of pass-through effect conditional on cointegration rank. Secondly, I show that the equilibrium measure of the pass-through effect may not be unique and discuss what equilibrium changes of other endogenous variable under analysis accompany different measures of exchange rate pass-through. These changes are related to the conventional long-run identification restrictions for identification of structural shocks commonly employed in the structural VAR literature.

To facilitate the analysis consider a system of variables (1) in which the effect of the nominal exchange rate on prices is analyzed taking into account the endogenous dynamics in, foreign prices, output and the interest rate differentials:

$$X_t = (p_t, e_t, p^*_t, i_t - i^*_t, y_t).$$

(1)

where $y_t$ denotes output, $e_t$ is the nominal exchange rate, $p_t$ are domestic prices, $p^*_t$ foreign prices and $i_t - i^*_t$ the nominal interest rate differential between the domestic and the foreign economy. Such a system of variables is considered also in the empirical analysis of this paper and is motivated by standard macroeconomic theory. Domestic prices, the nominal exchange rate and foreign prices are related through the (relative) purchasing power relation. The interest rate differential affects exchange rate dynamics through the interest rate parity. Finally, output captures the effect of real factors on all endogenous variables in the system. Moreover, it is typically of interest to evaluate also the effect of exchange rate changes on output.

The variables are modelled as a cointegrated VAR

$$\Delta X_t = \Pi X_{t-1} + \sum_{i=1}^{k-1} \Gamma_i \Delta X_{t-i} + \Phi D_t + \epsilon_t$$

(2)

with a corresponding reduced rank condition $\Pi = \alpha \beta'$ (see Johansen (1995) for a detailed presentation). The matrix $\beta$ contains the cointegrating relations and $\alpha$ contains the corresponding loading coefficients. The cointegration coefficient between the exchange rate and prices is of central interest because we need establish under what conditions it can be interpreted as equilibrium pass-through effect. A cointegration relation containing $e_t$ and $p_t$ can be generically written in regression format as follows

$$p_t = \beta_1 e_t + \beta_2 p^*_t + \beta_3 (i_t - i^*_t) + \beta_4 y_t + \text{error}_t$$

(3)

Note that depending on cointegration rank all variables need not enter such a cointegration relation. With cointegration rank equal to one the relation would be as in (3). With cointegration rank higher than one at least on of the coefficients $\beta_2$ to $\beta_4$ must be zero. In any case we need to check whether $\lambda_1$ can be interpreted as equilibrium pass-through effect. In other words, we need to check whether, based on an estimate of the cointegration relation of the type (3), we can say that with a long-run change in $e_t$ by 1 percent prices change by $\beta_1$ percent in equilibrium. This is a question about identification of the pass-through effect.

To establish the conditions for identification I invoke results of Johansen (2002). From the solution of the error-correction model it follows that the long-run value $X_{\infty/t}$ as a function of current values $(X_t, X_{t-1}, ..., X_{t-k+1})$ is given by

$$X_{\infty/t} = \lim_{h \to \infty} E \left( X_{t+h} \mid X_t, X_{t-1}, ..., X_{t-k+1} \right) = C \left( X_t - \sum_{i=1}^{k-1} \Gamma_i X_{t-i} \right)$$

(4)
where matrices $C$ and $\Gamma$ are defined as

$$C = \beta_\perp (\alpha'_\perp \Gamma \beta_\perp)^{-1} \alpha'_\perp$$

and $\alpha_\perp$ and $\beta_\perp$ are the orthogonal complements to $\alpha$ and $\beta$ respectively. Thus, it follows from the definition of matrix $C$ that the long-run changes in endogenous variables are proportional to $\beta_\perp$. A given long-run change $k \in \text{sp}(\beta_\perp)$ can be achieved either by adding $k$ to all current values or by adding $\Gamma k$ to $X_t$ - a short-run change. It is also assumed in what follows that cointegration vectors are identified using zero restriction, which justifies the interpretations used below (see Proposition 2 in Johansen, 2002). The following proposition gives a sufficient and necessary condition for identification of the pass-through effect.

**Proposition 1.** Equilibrium pass-through effect is identified if and only if the cointegration rank $r$ is equal to 1 plus the number of variables other than domestic prices with a non-zero coefficient in $k \in \text{sp}(\beta_\perp)$.

**Proof.** $X_t$ is a $p_t$ dimensional vector of variables. Without loss of generality assume $p_k$ is placed first and $e_t$ second. Consider the following vector of long-run changes: $k = (\lambda, 1, \mu, 0_{1 \times (p_k - 2 - n)})$, that is, a long-run change in the nominal exchange rate by one unit accompanied by a long-run change in prices by $\lambda$ units, while allowing for a non-zero effect on $n$ variables in sub-vector $\mu$. $\lambda$ measures the equilibrium pass-through effect and is a parameter that needs to be uniquely identified. Note that $k \in \text{sp}(\beta_\perp)$, hence the parameters in $k$ must solve $k'\beta = 0$. $\beta$ is rxp and it must be identified using zero restriction (see Johansen, 2002). $k$ has $n + 1$ unknown parameters. $k'\beta = 0$ is therefore a system of $r$ linear equations and $n + 1$ unknowns. It has a unique solution when $r = 1 + n$. In such a case $\lambda$ is uniquely identified. Unless we have some prior statistically non-testable information for parameters of $\mu$ this is also the only case when it can be identified from the estimated cointegration coefficients. $\blacksquare$

It directly follows from Proposition 1 that the identification of pass-through effect implies also that a long-run equilibrium change in the exchange rate has a non-zero equilibrium effect on $n = r - 1$ variables in $X_t$. This leads to the following corollary to Proposition 1.

**Corollary 2.** When pass-through effect is identified in a $p$-dimensional system, permanent exchange rate changes are associated with a non-zero equilibrium changes in exactly $r - 1$ variables other than domestic prices $p_k$.

Proposition 1 and its corollary have important economic implications since they imply that conditional on cointegration rank one can obtain the equilibrium exchange rate pass-through on prices can be identified from cointegration coefficients, but permanent exchange rate changes are associated with different effects on the remaining variables of the system. For a better illustration of the identification issue let us consider the following example. In the system of variables (1) genuine cointegration is found for ranks 1 to 4. With rank 1 a single cointegrating relation can be conveniently written in regression format as (3) which corresponds to cointegrating vector $\beta = (1, -\beta_1, -\beta_2, -\beta_3, -\beta_4)'$. As discussed above we need to check whether $\beta_1$ can be interpreted as a measure of pass-through effect. Consider a long-run change $k = (\beta_1, 1, 0, 0, 0)'$, that is, a long-run change in prices by $\beta_1$ percent accompanied by a long-run change in the exchange rate by 1 percent while leaving foreign prices, the interest rate spread and real output unchanged. Note that $k'\beta = 0$. In such a case $k \in \text{sp}(\beta_\perp)$ and we could interpret $\beta_1$ as the equilibrium pass-through effect that is consistent with a zero equilibrium effect on the remaining variables of the system.

If we want to consider an equilibrium change with a non-zero equilibrium effect on the remaining variables in the system and see if we can interpret $\beta_1$ as the pass-through effect we need to explore the feasibility of the long-run change of the form $k = (\lambda, 1, \mu, 0, 0)'$. Clearly such a vector is not orthogonal to $\beta$ for any $\mu \neq 0$ and hence $\beta_1$ cannot be interpreted as the pass-through effect. In such a case the equilibrium pass-through effect cannot be directly obtained from estimated cointegration coefficients.
Next consider the case with \( r = 2 \). Without loss of generality assume that cointegrating vectors are identified in such a way that there is a \( r \times r \) identity matrix as the upper block of \( \beta \). In regression format they can be written as

\[
\beta_1 = [1, 0, \beta_{13}, \beta_{14}, \beta_{15}]'
\]

\[
\beta_2 = [0, 1, \beta_{23}, \beta_{24}, \beta_{25}]'
\]

In line with Proposition 1 in such a case the equilibrium exchange rate pass-through can be identified from cointegration coefficients if we allow for an accompanying non-zero effect on one additional variable. This implies that we need to consider long-run changes of the type \( k = (\lambda, 1, \mu, 0, 0)' \) or \( k = (\lambda, 1, 0, \mu, 0)' \) or \( k = (\lambda, 1, 0, 0, \mu)' \), corresponding to non-zero long-run change in foreign prices, the interest rate spread and output respectively.

In either case, by solving \( k'\beta = 0 \) the equilibrium pass-through effect \( \lambda \) is identified and estimated by

\[
\lambda = \frac{\beta_{1j}}{\beta_{2j}}; \quad j = 3, 4, 5.
\]

With \( j = 3 \) we consider \( k = (\lambda, 1, \mu, 0, 0)' \) - a long-run change in the exchange rate, domestic and foreign prices that is typically considered in the analysis of the purchasing power parity hypothesis. From an economic point of view such a change can be considered as a consequence of purely nominal factors (shocks). With \( j = 4 \) we consider \( k = (\lambda, 1, 0, \mu, 0)' \) - a long-change the exchange rate, domestic prices and the interest rate spread. We can interpret such a change as coming from financial shocks and is associated by a different equilibrium exchange rate pass-through effect than in the case of a nominal shock. Finally, With \( j = 5 \) we consider \( k = (\lambda, 1, 0, 0, \mu)' \) - a long-run change in the exchange rate, domestic prices and real output \( y \), which comes about due to permanent real shocks with corresponding equilibrium exchange rate pass-through effect.

In the case of three cointegrating relations we would have

\[
\beta_1 = [1, 0, 0, \beta_{14}, \beta_{15}]'
\]

\[
\beta_2 = [0, 1, 0, \beta_{24}, \beta_{25}]'
\]

\[
\beta_3 = [0, 0, 1, \beta_{34}, \beta_{35}]'
\]

and the corresponding long-run changes, which according to Proposition 1 contain two variables in addition to the exchange rate and prices with non-zero coefficient. In particular, \( k = (\lambda, 1, \mu_1, \mu_2, 0)' \) or \( k = (\lambda, 1, 0, \mu_1, \mu_2)' \) or \( k = (\lambda, 1, \mu_1, 0, \mu_2)' \). Also in such a case, the estimated equilibrium exchange rate pass-through can be obtained directly from the cointegration coefficients by solving \( k'\beta = 0 \) for these long-run changes.

The largest possible cointegration rank in our example is 4. In such a case there is only one unit root in the system and hence \( \beta_\perp \) a one-dimensional space. Feasible long-run changes include non-zero effects on all modelled variables. The equilibrium exchange rate pass-through effect is in such case the simplest to determine as the four cointegration relations include two variables each. If we identify the cointegration space such that one of the relations includes the exchange rate and domestic prices, the equilibrium exchange rate is measured directly by the corresponding cointegration coefficient.

These examples reveal that the identification of long-run or equilibrium pass-through effect depends on cointegration rank. Depending on rank we can find different possibilities of the long-run changes of other modelled variables other than the exchange rate and prices. Clearly, such long-run changes need to be sensible also from economic point of view, otherwise we reduce the issue of estimating the equilibrium pass-through effect to a mechanical procedure with limited value for policy makers.

When identification can be achieved we can also determine the corresponding contemporaneous pass-through effect – a short-run change – that supports a given long-run change leading to a conventional impulse response analysis that I also consider in the next section. As seen above, a given long-run change \( k \in sp(\beta_\perp) \) can be achieved by adding \( \Gamma_k \) to \( X_t \). We can interpret this change also as the effects of shocks that clearly have permanent effects on variables in \( X_t \). In fact, this is a restriction on any type of shock in structural sense (real or nominal) that economic theory can justify to have a permanent change given by \( k \in sp(\beta_\perp) \).
2.1. Implications of identification problem for other methodological approaches

The analysis thus far focused on the interpretation of cointegration coefficients and consequently highlighted potential weaknesses in existing studies using the cointegrated VAR framework to study the equilibrium or long-run exchange rate pass-through. However, the discussion of the identification problem has some implications also for single-equation and SVAR-based studies of pass-through effect.

It is clear from the above discussion that a given single-equation estimate of

\[ p_t = \lambda_1(L)e_t = \lambda_2(L)p_t^* + \lambda_3(L)(i_t - i_t^*) + \lambda_3(L)y_t + \varepsilon_t \]

would not enable us to directly interpret \( \lambda_1 \) as the measure of pass-through effect. A ceteris paribus interpretation of \( \lambda_1 \) explicitly assumes that \( p_t^*, i_t - i_t^* \) and \( y_t \) do not change as \( e_t \) changes, which is hard to assume from an economic point of view.

In general, any specification of the empirical model for estimation of the pass-through effect that contains variables that are according to economic theory in equilibrium dynamically linked to the nominal exchange rate calls for systems estimation of the model, and within that model checking for the identification of the equilibrium effect. This reasoning goes beyond the estimation of only exchange rate pass-through. As follows from Johansen (2002), it is applicable to any type of empirical analysis where correct interpretation of cointegration coefficients is of importance.

SVAR analysis of pass-through effect commonly relies on identification of exchange rate shocks and estimation of impulse responses to an exchange rate shock. As is shown in the next section also the approach used in this paper can accommodate impulse response analysis and study the short-run pass-through effects that lead to long-run changes. Note, however, that there is one important difference in the approaches. While the SVAR approach necessarily uses non-testable (just)identifying restrictions, the approach proposed in this paper does not rely on such restrictions and uses only the estimated cointegration coefficients. In this respect it is partial-identification scheme that does not need to impose a full set of identifying restriction on the VAR parameter to deliver the impulse response analysis of the exchange rate pass-through.

3. Empirical application - Exchange rate pass-through in the Euro area

This section illustrates the procedure of the identification of the exchange rate pass-through for the case of the Euro area. The system of variables I consider is the same as the one used in the example of the previous section:

\[ X_t = (p_t, e_t, p_t^*, i_t - i_t^*, y_t), \]  

where \( p_t \) stands for the log of the Euro area consumer price index excluding energy and food prices, \( e_t \) is the log of the nominal effective exchange rate, \( p_t^* \) is the log of the world GDP deflator, \( i_t - i_t^* \) is the spread between the short-term interest rate in the Euro area and the US 3-month T-bills and \( y_t \) is the log of the Euro area GDP. The Euro area data come from the 2013 update of the Euro Area Wide Model dataset of Fagan et al. (2001). They are quarterly and cover the period 1990-2012.\(^3\) The US data are taken from the FRED database.

The lag length of the system (2) has been chosen by complementary use of standard information criteria and the usual Wald-type tests for lag reduction. It proved sufficient to include two endogenous lags, which delivers also a statistically well specified model (see upper panel of Table 1). The test for cointegration rank in Table 1 indicates rank 2, which is consistent with the evidence of three large roots in the system.

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\(^3\)The data can be downloaded from the Euro area business cycle network webpage (www.eabcn.org/area-wide-model).
Table 1: Multivariate misspecification tests and characteristic roots and trace tests for the I(1) systems

<table>
<thead>
<tr>
<th></th>
<th>Res. autocorr. 1</th>
<th>Res. autocorr. 1-4</th>
<th>Normality</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$F(25, 261) = 1.42$</td>
<td>$F(100, 272) = 1.18$</td>
<td>$\chi^2(10) = 65.34$</td>
</tr>
<tr>
<td></td>
<td>$p$-val = 0.09</td>
<td>$p$-val = 0.15</td>
<td>$p$-val = 0.00</td>
</tr>
<tr>
<td>Modulus of 6 largest characteristic roots</td>
<td>0.99 0.93 0.93 0.89 0.89 0.55</td>
<td>0.99 0.93 0.93 0.89 0.89 0.55</td>
<td>0.99 0.93 0.93 0.89 0.89 0.55</td>
</tr>
<tr>
<td>Trace test</td>
<td>3.94 10.64 24.13 48.36 82.49</td>
<td>4 3 2 1 0</td>
<td>4 3 2 1 0</td>
</tr>
<tr>
<td>$p$-value</td>
<td>0.05 0.34 0.20 0.04 0.00</td>
<td>0.05 0.34 0.20 0.04 0.00</td>
<td>0.05 0.34 0.20 0.04 0.00</td>
</tr>
</tbody>
</table>

Table 2: Estimated Cointegration Relations and Loading Coefficients

<table>
<thead>
<tr>
<th></th>
<th>$\beta_1$</th>
<th>$\beta_2$</th>
<th>$\alpha_1$</th>
<th>$\alpha_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$p_t$</td>
<td>1.00</td>
<td>-</td>
<td>-0.03</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.01)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$e_t$</td>
<td>-</td>
<td>1.00</td>
<td>0.11</td>
<td>0.01</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.29)</td>
<td>(0.02)</td>
</tr>
<tr>
<td>$p_t^*$</td>
<td>-1.18</td>
<td>-0.02</td>
<td>0.13</td>
<td>-0.01</td>
</tr>
<tr>
<td></td>
<td>(0.17)</td>
<td>(2.11)</td>
<td>(0.05)</td>
<td>(0.00)</td>
</tr>
<tr>
<td>$i_t - i_t^*$</td>
<td>0.01</td>
<td>0.14</td>
<td>-20.15</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>(0.00)</td>
<td>(0.03)</td>
<td>(5.17)</td>
<td>(0.45)</td>
</tr>
<tr>
<td>$y_t$</td>
<td>-0.17</td>
<td>-0.33</td>
<td>-0.07</td>
<td>-0.00</td>
</tr>
<tr>
<td></td>
<td>(0.13)</td>
<td>(1.65)</td>
<td>(0.05)</td>
<td>(0.00)</td>
</tr>
</tbody>
</table>

Notes: Standard errors in parentheses.

The left panel of Table 1 presents the estimates of cointegration vectors. The right panel reports the corresponding adjustment coefficients. The cointegration coefficients are statistically identified by means of a leading identity matrix. Such an approach is convenient for the computation of the equilibrium exchange rate pass-through effect as demonstrated in the previous section.

Before proceeding to the discussion of how the equilibrium exchange rate pass-through can be estimated in the current empirical example it useful to consider an example that demonstrates potential pitfalls in interpreting cointegration coefficients. It can be shown in the present case that one cannot reject the hypothesis that the following cointegration vector lies in the cointegration space

$$\tilde{\beta} = [1, -1, -1, -0.13, 0.06]'$$

The likelihood ratio statistic of the test of the hypothesis that the second and the third element of this vector are equal to $-1$ is equal to 0.004 with corresponding $p$-value of 0.95. This implies that we cannot reject the hypothesis that one of the cointegration vectors can be written as

$$p_t = e_t + p_t^* + 0.13 (i_t - i_t^*) - 0.06 y_t + error_t$$

Because the coefficients corresponding to the nominal exchange rate and the foreign price level are equal to one, a typical *ceteris paribus* interpretation of such a cointegration relation would be that the equilibrium exchange rate pass-through effect is perfect and, in addition, the purchasing power parity holds.

Note, however, that such an interpretation would be false. For the purchasing power parity to hold we need to consider a long-run change of the type $\tilde{k} = [1, 1, 1, 0, 0]$, i.e. domestic prices,
foreign prices and the exchange rate moving one-to-one in the long run. But this is clearly not the case. It is evident that $k \beta = -1 \neq 0$, and hence neither the purchasing power parity nor the perfect pass-through assumptions hold.

From the estimates of cointegration coefficients in Table 2 we can estimate also the degrees of equilibrium exchange rate pass-through effect. Given that cointegration rank is set to two the discussion in Section 2 suggests that the equilibrium exchange rate pass-through is estimated as

$$\lambda = \frac{\beta_1}{\beta_{2j}}, \quad j = 3, 4, 5.$$ 

conditional on whether we consider the accompanying long-run change in either foreign prices or the interest rate spread or the Euro area prices. In line with the discussion of the previous section we will label these changes as coming from nominal, financial and real permanent shocks respectively. It is worth remembering that these changes are required for identification of the equilibrium pass-through effect from the estimated cointegration coefficients reported in Table 2.

The estimates of the equilibrium pass-through effects are reported in Table 3. As we can observe they differ quite significantly depending on which variable other than domestic prices is allowed to change in equilibrium with the nominal exchange rate. To facilitate the interpretation of the differences in the pass-through effects Figures 1 - 3 report the corresponding impulse response functions that deliver the long-run changes and co-movements between prices and the exchange rate reported in Table 3.

The value of the equilibrium pass-through effect conditional on a nominal shock is surprising at first sight as it turns out to be highly negative. The results can be rationalized, however, by looking at the impulse responses in Figure 1 that are obtained by simulating the effect of an initial change in $X_t$ by $\Gamma k$.

We can observe that the nominal shock drives prices permanently down both in the Euro area and the rest of the world by practically the same amount. The adjustment is faster for the Euro area prices, which is consistent with a positive response of the interest rate spread. The corresponding change in the exchange rate is in principle allowed to be non-zero and from an initial negative response becomes only slightly and insignificantly positive. Given an almost parallel trajectory of prices, leaving the real exchange rate unchanged, weak adjustment through the nominal exchange rate is not surprising, and neither is an odd measure of equilibrium exchange rate pass-through effect.

Table 3: Long-run changes and equilibrium exchange rate pass-through effect

<table>
<thead>
<tr>
<th>Long-run shock</th>
<th>Equilibrium pass-through</th>
<th>Perfect pass-through test</th>
<th>$\chi^2$(1)</th>
<th>p-val</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nominal</td>
<td>-2.99</td>
<td>*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Financial</td>
<td>0.10</td>
<td>4.71</td>
<td>0.03</td>
<td></td>
</tr>
<tr>
<td>Real</td>
<td>0.35</td>
<td>0.58</td>
<td>0.44</td>
<td></td>
</tr>
</tbody>
</table>

* no convergence in estimation algorithm

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4The 90% confidence intervals are obtained by bootstrap.
Conditional on a financial shock that permanently affects the interest rate spread the estimated equilibrium exchange rate pass-through is positive, but low. A 1 percent equilibrium increase in the effective exchange rate is associated with a 0.1 percent increase in the Euro area prices. Low equilibrium pass-through effect is confirmed by a formal test of perfect equilibrium pass-through, i.e. $\lambda = 1$. This would obtain if $\beta_{14} = \beta_{24}$, which is also the hypothesis tested by a likelihood ratio test. The test clearly rejects the perfect pass-through hypothesis.

The corresponding impulse responses in Figure 2 confirm the interpretation of long-run changes as coming from permanent financial shocks. The interest rate spread is permanently driven down, which immediately weakens the euro. Domestic prices quickly rise to the new long-run equilibrium, while output reacts only temporarily.

Finally, we turn to a real shock, a long-run change that permanently changes real GDP. In such a case the equilibrium pass-through effect is the highest, with the point estimated of 0.35. Moreover, a formal test cannot reject the hypothesis $\beta_{15} = \beta_{25}$, which implies a perfect equilibrium pass-through.

The impulse responses in Figure 3 show that in response to a permanent increase in the Euro area GDP that is accompanied by a permanent depreciation of the effective exchange rate (induced by a temporary reduction in the interest rate spread) domestic prices gradually rise to the new equilibrium level. Relative to the case of the financial shock, the exchange rate pass-through is slower in the short-run but higher in the long run.

The interpretation requires of bit of caution as the estimated responses of the exchange rate and domestic prices results to be rather imprecisely estimated and consequently not statistically different from zero already at short horizons. This feature partially explains the results of the test for a perfect pass-through effect in Table 3.
Figure 2: Impulse responses to a permanent financial shock

Figure 3: Impulse responses to a permanent real shock
4. Concluding discussion

The purpose of the paper is to contribute to the literature on exchange rate pass-through estimation from methodological and empirical point of view. The empirical analysis is performed within the cointegrated VAR framework that in principle enables the estimation of the equilibrium pass-through effect of nominal exchange rate changes on prices. As shown by Johansen (2002), this requires a proper interpretation of cointegration coefficients, which, combined with economic theory, in the present framework translates into the problem of identification of the equilibrium pass-through effect. Systems of variables for pass-through estimation must contain all crucial variables that account for complex macroeconomic interdependence in open economies. In such a case, we cannot discuss equilibrium effects of a one percentage point change in nominal exchange rate on prices without taking into account also the endogenous effect on other variables. This implies that identification cannot be automatically achieved for all orders of cointegration rank. Moreover, each possible value of cointegration rank leads to different conclusions about the equilibrium effects of exchange rate changes.

The process of identification of equilibrium pass-through is demonstrated on the Euro area data. It is shown that the equilibrium exchange rate pass-through is incomplete, but lower conditional on the permanent financial shocks and higher conditional on permanent increases in real output. For pure nominal price shock the pass-through effect may not even turn out to be positive.

These results offer a policy implication related to the expansive unconventional monetary policy measures introduced by the ECB in late 2014. These are expected to depreciate the euro relative to other major world currencies and consequently stimulate economic activities and curb deflationary pressures. From an economic point of view these unconventional policy measures are most closely related to the case of a permanent financial shock. The results in this paper show that the effects of a permanently weaker euro on economic activity is rather limited and it has a weak positive effect on the Euro area consumer prices.
References


