OPTIMAL INVENTORY CONTROL WITH ADVANCE SUPPLY INFORMATION

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ABSTRACT: *It has been shown in numerous situations that sharing information between the companies leads to improved performance of the supply chain. We study a positive lead time periodic-review inventory system of a retailer facing stochastic demand from his customer and stochastic limited supply capacity of the manufacturer supplying the products to him. The consequence of stochastic supply capacity is that the orders might not be delivered in full, and the exact size of the replenishment might not be known to the retailer. The manufacturer is willing to share the so-called advance supply information (ASI) about the actual replenishment of the retailer's pipeline order with the retailer. ASI is provided at a certain time after the orders have been placed and the retailer can now use this information to decrease the uncertainty of the supply, and thus improve its inventory policy. For this model, we develop a dynamic programming formulation, and characterize the optimal ordering policy as a state-dependent base-stock policy. In addition, we show some properties of the base-stock level. While the optimal policy is highly complex, we obtain some additional insights by comparing it to the state-dependent myopic inventory policy. We conduct the numerical analysis to estimate the influence of the system parameters on the value of ASI. While we show that the interaction between the parameters is relatively complex, the general insight is that due to increasing marginal returns, the majority of the benefits are gained only in the case of ull, or close to full, ASI visibility.*

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1. Introduction

Nowadays companies are facing difficulties in effectively managing their inventories mainly due to the highly volatile and uncertain business environment. While they are trying their

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best to fulfill the demand of their customers by using more or less sophisticated inventory control policies, their efforts can be severely hindered by the unreliable and limited deliveries from their suppliers. Due to the widespread trend of establishing plants overseas or outsourcing to specialists it has become increasingly more difficult for the companies to retain control over their procurement process. As the complexity of supply networks grows, so do the challenges and inefficiencies the companies are facing; orders get lost or are not delivered in full, shipments are late or don't arrive at all.

It has been well acknowledged both in the research community as well as by practitioners that these uncertainties can be reduced and better supply chain coordination can be achieved through the improved provision of information (Lee and Padmanabhan 1997, Chen 2003). While the performance of a supply chain depends critically on how its members coordinate their decisions, sharing information can be considered as the most basic form of coordination in supply chains (Austin et al. 1998, Manrodt et al. 2005). In light of this, the concept of achieving a so-called Supply Chain Visibility is gaining on importance, as it provides accurate and timely information throughout the supply chain processes and networks (Rassameethes et al. 2000, Jahre et al. 2007, Barratt and Adegoke 2007). This enables companies to share the information through often already established B2B communication channels and ERP solutions. EDI formatted electronic notifications on the status of the order fulfillment process, such as order acknowledgements, inventory status, Advance Shipment Notices (ASN), and Shipment Status Messages (SSM) are shared, enabling companies to track and verify the status of their order and consequently foresee supply shortages before they happen (Choi 2010). There are also multiple examples of companies like UPS, FedEx and others in shipping industry, and Internet retailers like eBay and Amazon, that are offering real time order fulfillment information also on the B2C level.

Real visibility in the supply chain can be regarded as a prerequisite for the companies to reach new levels of operating efficiency, service capabilities, and profitability for suppliers, logistics partners, as well as their customers. However, while the technological barriers to information sharing are being dismantled, the abundance of the available information by itself is not a guarantee for improved performance. Therefore the focus now is on developing new tools and technologies that will use this information to improve the current state of the inventory management practices.

In this paper, we investigate the benefits of advance supply information (ASI) sharing. We consider a retailer facing stochastic demand from the end customers and procuring the products from a single manufacturer with stochastic limited supply capacity. We assume that the order is replenished after a given fixed lead time, which constitutes of order processing, production and shipping delay. However it can happen that the quantity received by the retailer is less than what he ordered originally. This supply uncertainty can be due to, for instance, the allocation policy of the manufacturer, which results in variable capacity allocations to her customers or to an overall capacity shortage at certain times. This stochastic nature of capacity itself may be due to multiple causes, such as variations in the workforce (e.g. holiday leaves), unavailability of machinery or multiple products sharing the total capacity.

We assume that the manufacturer tracks the retailer's order evolution and at certain point, when she can assess the extent to which the order will be fulfilled, she shares ASI with the retailer, giving him feedback on the actual replenishment quantity ahead of the time of the physical delivery of products. ASI enables the retailer to respond to the possible shortage by adjusting his future order decisions, and by doing this possibly offset the negative impact of the shortage. Based on this rationale we pose the following two research questions: (1) How can we integrate ASI into inventory decision model, and subsequently characterize the optimal policy? (2) Can we quantify the value of ASI and establish the system settings where utilizing ASI is of most importance?

The practical setting in which the above modeling assumptions could be observed is food processing industry, where the food processing facilities/manufacturers are being supplied with the agricultural products. The products are harvested periodically and the product availability is changing through time depending on a variety of factors: weather, harvesting capacity, etc. Also it is reasonable to assume that supply capacity cannot be backordered as harvested products cannot be stored for longer periods. Khang and Fujiwara (2000) discuss this scenario for the frozen seafood industry, however they assume that the retailers' orders are fulfilled immediately by the manufacturer. We believe it is more realistic that the supply process is taking a number of time periods, more so that the process can be broken into two phases. As the order is made by the retailer, the harvesting part of the production process is underway, where the production outcome is uncertain. Then the products are delivered to the food processing facility. At this point the product availability is revealed and is no longer uncertain, and ASI is communicated to the customer in the form similar to ASN. The actual replenishment follows after the product is fully processed. This fully processed product can now also be stored.

Our work builds on the broad research stream of papers assuming uncertainty in the supply processes. In the literature the supply uncertainty is commonly attributed to one of the two sources: yield randomness and randomness of the available capacity. Our focus lies within the second group of problems, where Federgruen and Zipkin (1986a,b) were the first to address the capacitated stationary inventory problem with a fixed capacity constraint and have proven the optimality of the modified base-stock policy. Kapuscinski and Tayur (1998) extend this result by studying the non-stationary version of the model, where they assume periodic demand. Later, a line of research extends the focus to capture the uncertainty in capacity, by analyzing models with limited stochastic production capacity (Ciarallo et al. 1994, Güllü et al. 1997, Khang and Fujiwara 2000, Iida 2002). Ciarallo et al. (1994) explore different cases of the stochastic capacity constraint in a single and multiple periods setting. In the analysis of a single period problem, they show that stochastic capacity does not affect the order policy. The myopic policy of newsvendor type is optimal, meaning that the decision maker is not better off by asking for a quantity higher than that of the uncapacitated case. For a finite horizon stationary inventory model they show that the optimal policy remains to be a base-stock policy, where the optimal base-stock level is increased to account for the possible, however uncertain, capacity shortfalls in the future periods. Iida (2002) extends this result for the non-stationary environment.

Although a lot of attention in recent decades has been put in assessing the benefits of sharing information in the supply chains, the majority of the research is focused on studying the effect of sharing downstream information, in particular demand information (Gallego and Özer 2001, Karaesmen et al. 2003, Wijngaard 2004, Tan et al. 2007, Özer and Wei 2004). Review papers by Chen (2003), Lau (2007) and Choi (2010) show that sharing upstream information has been considered in the literature in the form of sharing lead time information, production cost information, production yield, and sharing capacity information. It has been shown by numerous researchers that information sharing decreases the bullwhip effect (the increasing variance of orders in a supply chain), however it was also shown that despite being optimal, the base-stock policy is an instigator of increased order variability (Jakšič and Rusjan 2008).

Capacity information sharing is of particular interest to our paper, where several papers have been discussing sharing information on future capacity availability (Jakšič et al. 2011, Altuğ and Muharremoğlu 2011, Çinar and Güllü 2012, Atasoy et al. 2012). Jakšič et al. (2011) study the benefits of sharing perfect information on future supply capacity available for orders to be placed in future periods. They show that the optimal ordering policy is a state-dependent base-stock policy characterized by the base-stock level that is a function of advance capacity information. Altuğ and Muharremoğlu (2011) work on a similar model; however they assume that the evolution of the capacity availability forecasts is done via the Martingale Method of Forecast Evolution (MMFE). The main difference in the way information is shared in the above cases compared to sharing ASI in this paper lies is the assumption about the time delay between the placement of the order and the time the information on the available supply capacity is revealed. In our case, the supply capacity information is revealed after the order has been placed and the lack of supply capacity availability results in the replenishment below the initial order. In the case of information about future capacity availability the order is aligned with this availability, and thus replenished in full. ASI thus only allows the decision maker to respond to the actual realized shortages in a more timely manner. While in the case of information on future capacity availability, the decision maker can anticipate the potential future shortages and accordingly adopt his ordering strategy. While in the latter case, the savings potential is higher, it is reasonable to assume that ASI is likely to be more reliable and easier to obtain in a practical setting.

Zhang et al. (2006) discuss the benefits of sharing advance shipment information. A setting in which a company receives the exact shipment quantity information is closely related to the one proposed in this paper, however they assume that inventory is controlled through a simple non-optimal base-stock policy and as such it fails to capture the uncertainty of supply. Our model can be considered as a generalization of the model by Zhang et al. (2006), as we allow for both, demand and supply capacity to be stochastic, and more importantly we model the optimal system behavior by considering considering the optimal inventory policy that is able to account for the supply uncertainty by setting appropriate safety stock levels. We propose that by having timely feedback on actual replenishment quantities through ASI, we can refine the inventory policy and improve its performance. To our knowledge the exploration of the relationship between the proposed way of modeling ASI and the optimal policy parameters has not yet received any attention in the literature.

Our contributions in this study are twofold. The focus is on modeling a periodic review single-stage inventory model with stochastic demand and limited stochastic supply capacity with the novel feature of improving the performance of the inventory control system through the use of ASI. Despite a relatively simple and intuitive structure of the optimal policy, the major difficulty lies in determining the optimal base-stock levels to which the orders should be placed. Already for the single-stage model under consideration, we need to resort to the numerical analysis to estimate these. Even more so, analyzing the real-life supply chains inevitably leads to the complex system state description, causing the state space to become large and eventually too large to evaluate all possible future scenarios or realizations. The problem commonly referred to as *"Curse of dimensionality"* (Puterman 1994). This greatly reduces the likelihood that a realistic inventory problem can be solved. One way to tackle this problem is to search for the approximate inventory policy, which comes at the cost of suboptimal performance. In our case we opt to analyze the myopic (shortsighted) inventory policy, and compare its parameters to the optimal ones.

In addition to the analytical and numerical results, we provide some relevant managerial insights related to optimal inventory control and the value of information sharing between the supply chain parties. The main dilemma in stochastic inventory management revolves around setting the appropriate safety stock levels, where the performance of the inventory system will depend on finding the right trade-off between the costs of holding the safety stocks and achieving the desired service level to the customers. While it is unrealistic that the companies would be able to integrate the proposed optimal policy into their ERP system, we provide some general guidelines on how the safety stock levels are influenced by the demand and supply uncertainty, and motivation for the companies to stimulate the information exchange with their supply chain partners.

The remainder of the paper is organized as follows. We present a model incorporating ASI and its dynamic cost formulation in Section 2. The optimal policy and its properties are discussed in Section 3. We proceed by the study of the approximate inventory policy based on the state-dependent myopic policy in Section 4. In Section 5 we present the results of a numerical study and point out additional managerial insights. Finally, we summarize our findings and suggest directions for future research in Section 6.

2. Model formulation

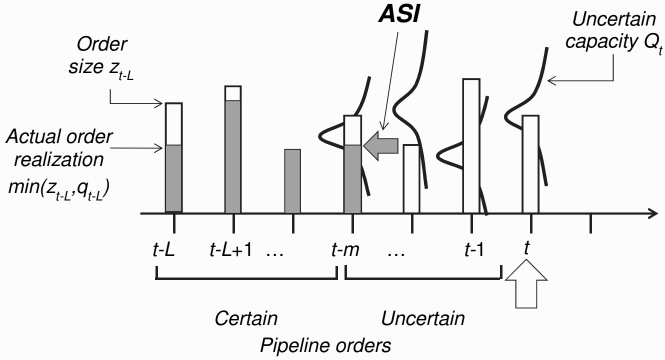
In this section, we introduce the notation and the model of advance supply information for orders that were already placed, but are currently still in the pipeline. The model under consideration assumes periodic-review, stationary stochastic demand, limited stationary stochastic supply with fixed supply lead time, finite planning horizon inventory control system. Unmet demand is fully backlogged. However, the retailer is able to obtain ASI on supply shortages affecting the future replenishment of the orders in the pipeline from the manufacturer. We introduce the ASI parameter m that represents the time delay in which ASI is communicated with the retailer. The parameter m effectively denotes the number of periods between the time the order has been placed with the manufacturer and the time ASI is revealed. More specifically, ASI on the order z_{t-m} placed m periods ago is revealed in period t after the order z_t is placed in the current period (Figure 1). Depending on the available supply capacity q_{t-m}, ASI reveals the actual replenishment quantity, determined as the minimum of the two, $\min(z_{t-m}, q_{t-m})$. We assume perfect ASI. Observe that the longer the m, the larger is the share of the pipeline orders for which the exact replenishment is still uncertain. Furthermore, we assume that the unfilled part of the retailer's order is not backlogged at the manufacturer, but it is lost. We give the summary of the notation in Table 1 and we introduce some later upon need.

Table 1: Summary of the notation

T	:	number of periods in the finite planning horizon
L	:	constant nonnegative supply lead time, a multiple of review periods ($L \geq 0$);
m	:	advance supply information parameter, $0 \leq m \leq L$
h	:	inventory holding cost per unit per period
b	:	backorder cost per unit per period
α	:	discount factor ($0 \leq \alpha \leq 1$)
x_t	:	inventory position in period t before ordering
y_t	:	inventory position in period t after ordering
$\tilde{x}_t$	:	starting on-hand inventory in period t
z_t	:	order size in period t
D_t	:	random variable denoting the demand in period t
d_t	:	actual demand in period t
Q_t	:	random variable denoting the available supply capacity at time t
q_t	:	actual available supply capacity limiting order z_t given at time t, for which ASI is revealed m periods later

We assume the following sequence of events. (1) At the start of period t, the decision maker reviews the current inventory position x_t. (2) The ordering decision z_t is made up to uncertain supply capacity and correspondingly the inventory position is raised to $y_t = x_t + z_t$. (3) Order placed in period $t - L$ is replenished in the extent of $\min(z_{t-L}, q_{t-L})$, depending on the available supply capacity. ASI on the order placed in period $t - m$ is revealed, which enables the decision maker to update the inventory position by correcting it downward in the case of insufficient supply capacity, $y'_t = y_t - (z_{t-m} - q_{t-m})^+$, where $(x)^+ = max(x, 0)$.

Figure 1: Advance supply information.



(4) At the end of the period previously backordered demand and demand d_t are observed and satisfied from on-hand inventory; unsatisfied demand is backordered. Inventory holding and backorder costs are incurred based on the end-of-period on-hand inventory.

Due to positive supply lead time, each of the orders remains in the pipeline stock for L periods. For orders placed m periods ago or earlier we have already obtained ASI, while for more recent orders the supply information is not available yet. Therefore we can express the inventory position before ordering x_t as the sum of net inventory and the certain and uncertain pipeline orders:

$$x_t = \tilde{x}_t + \sum_{s=t-L}^{t-m-1} \min(z_s, q_s) + \sum_{s=t-m}^{t-1} z_s. \quad (1)$$

Note, that due to perfect ASI the inventory position x_t reflects the actual quantities that will be replenished for the orders for which ASI is already revealed, while there is still uncertainty in the actual replenishment sizes for recent orders for which ASI is not known yet.

Observe also that m denotes the number of uncertain pipeline orders. Therefore, m lies within $0 \leq m \leq L$, and the two extreme cases can be characterized as:

- $m = L$, or so-called *"No information case"*, which corresponds to the most uncertain setting as the actual replenishment quantity is revealed no sooner than at the moment of actual arrival. This setting is a positive lead time generalization of the Ciarallo et al. (1994) model.

- $m = 0$, or so-called *"Full information case"*, which corresponds to the full information case, where before placing the new order, we know the exact delivery quantities for all pipeline orders. This is the case with the least uncertainty within the context of our model. Observe however that the current order is still placed up to uncertain supply capacity.

When moving from period t to $t+1$, we obtain ASI for the order z_{t-m} placed in period $t-m$. Correspondingly, the inventory gets corrected downwards if the order exceeds the available supply capacity, thus inventory position x_t is updated in the following manner:

$$x_{t+1} = x_t + z_t - (z_{t-m} - q_{t-m})^+ - d_t. \tag{2}$$

Note, that there is dependency between the order quantity and the size of the correction of x_t. If z_t is high, it is more probable that the available supply capacity will restrict the replenishment of the order, thus the correction will be bigger, and vice versa for low z_t. To fully describe the system behavior, we do not only need to keep track of x_t, but also have to track the pipeline orders for which we do not have ASI yet. We denote the stream of uncertain pipeline orders with the vector $\vec{z}_t = (z_{t-m}, z_{t-m+1}, \ldots, z_{t-2}, z_{t-1})$. In period $t+1$, $\vec{z}_{t+1}$ gets updated by the inclusion of the new order z_t, and the order z_{t-m} is dropped out as its uncertainty is resolved through the received ASI.

A single period expected cost function is a function of x_t and all uncertain orders, including the most recent order z_t, given in period t. Cost charged in period $t+L$, $\tilde{C}_{t+L}(\tilde{x}_{t+L+1})$, reassigned to period t when ordering decision is made, can be expressed as:

$$C_t(y_t, \vec{z}_t, z_t) = \alpha^L E_{\vec{Q}_t, Q_t, D_t^L} \tilde{C}_{t+L}(y_t - \sum_{s=t-m}^{t} (z_s - Q_s)^+ - D_t^L), \tag{3}$$

where the expected inventory position after ordering (accounted for the possible future supply shortages), $E_{\vec{Q}_t, Q_t}(y_t - \sum_{s=t-m}^{t} (z_s - Q_s)^+)$, is used to cover the lead time demand, $D_t^L = \sum_{s=t}^{t+L} D_s$.

The minimal discounted expected cost function, optimizing the cost over a finite planning horizon T, from time t onward, and starting in the initial state $(x_t, \vec{z}_t)$, can therefore be written as:

$$f_t(x_t, \vec{z}_t) = \min_{x_t \le y_t} \{C_t(y_t, \vec{z}_t, z_t) + \alpha E_{D_t, Q_{t-m}} f_{t+1}(y_t - (z_{t-m} - Q_{t-m})^+ - D_t, \vec{z}_{t+1})\}, \text{ for } t \le T, \tag{4}$$

where $f_{T+1}(\cdot) \equiv 0$. The cost function f_t is a function of inventory position before ordering and orders given in last m periods, for which ASI has not yet been revealed.

3. Analysis of the optimal policy

In this section, we show the necessary convexity results of the relevant cost functions. This allows us to establish the structure of the optimal policy and show some of its properties.

Lets define J_t as the cost-to-go function of period t:

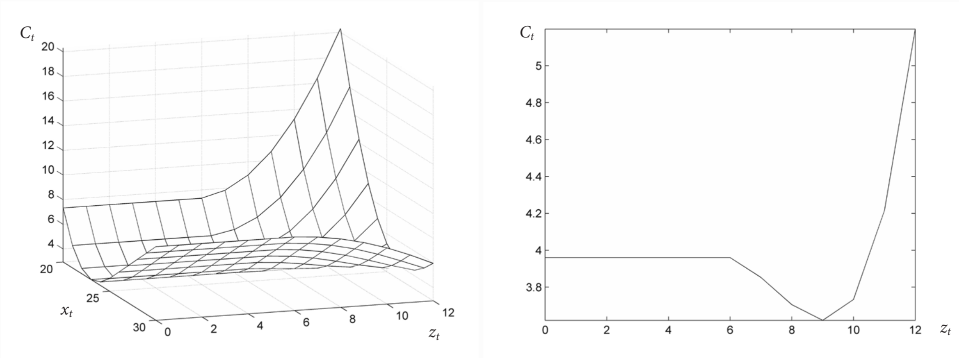
$$J_t(y_t, \vec{z}_t, z_t) = C_t(y_t, \vec{z}_t, z_t) + \alpha E_{D_t, Q_{t-m}} f_{t+1}(y_t - (z_{t-m} - Q_{t-m})^+ - D_t, \vec{z}_{t+1})\}, \text{ for } t \leq T. \quad (5)$$

The minimum cost function f_t defined in (4) can now be expressed as:

$$f_t(x_t, \vec{z}_t) = \min_{x_t \leq y_t} J_t(y_t, \vec{z}_t, z_t), \text{ for } t \leq T, \quad (6)$$

We proceed by establishing the necessary convexity results that allow us to establish the structure of the optimal policy. Observe that the single period cost function $C_t(y_t, \vec{z}_t, z_t)$ is not convex already for the zero lead time case as was originally shown by Ciarallo et al. (1994). $C_t(y_t, z_t)$ is shown to be convex in y_t and quasiconvex in z_t (Figure 2), which however still suffice for the optimal policy to exhibit the structure of the base-stock policy.

Figure 2: (a) $C_t(y_t, \vec{z}_t, z_t)$ as a function of x_t and z_t, and (b) $C_t(y_t, \vec{z}_t, z_t)$ as a function of z_t for a particular x_t.



We show that the results of the zero lead time case can be generalized to the positive lead time case, where the convexity of the costs functions in the inventory position is established given a more comprehensive system's state description $(x_t, \vec{z}_t)$. In Lemma 2 in the Appendix, we show that the single period cost function $C_t(y_t, \vec{z}_t, z_t)$ is not a convex function in general, but it exhibits a unique, although state-dependent, minimum. Based on this result one can show that the related multi-period cost functions $J_t(y_t, \vec{z}_t, z_t)$ and $f_t(x_t, \vec{z}_t)$ are convex in the inventory position y_t and x_t respectively (we show in Appendix that the convexity holds also for other characterizations of the inventory position), as shown in the next Lemma:

Lemma 1 *For any arbitrary value of information horizon m, value of the ASI vector $\vec{z}_t$ and the order z_t, the following holds for all t:*

1. *$J_t(y_t, \vec{z}_t, z_t)$ is convex in y_t,*
2. *$f_t(x_t, \vec{z}_t)$ is convex in x_t.*

Based on the results of Lemma 1, we establish a structure of the optimal policy in the following Theorem:

Theorem 1 *Assuming that the system is in the state $(x_t, \vec{z}_t)$, let $\hat{y}_t(\vec{z}_t)$ be the smallest minimizer of the function $J_t(y_t, \vec{z}_t, z_t)$. For any $\vec{z}_t$, the following holds for all t:*

1. *The optimal ordering policy under ASI is the state-dependent base-stock policy with the optimal base-stock level $\hat{y}_t(\vec{z}_t)$.*
2. *Under the optimal policy, the inventory position after ordering $y_t(x_t, \vec{z}_t)$ is given by*

$$y_t(x_t, \vec{z}_t) = \begin{cases} x_t, & \hat{y}_t(\vec{z}_t) \leq x_t, \\ \hat{y}_t(\vec{z}_t), & x_t < \hat{y}_t(\vec{z}_t). \end{cases} \tag{7}$$

The proof is by induction, where we provide the details in the Appendix. The optimal inventory policy is characterized by a single optimal base-stock level $\hat{y}_t(\vec{z}_t)$ that determines the optimal level of the inventory position after ordering. The optimal base-stock level however is state-dependent as it depends on uncertain pipeline orders $\vec{z}_t$, for which ASI has not yet been revealed. Observe that due to not knowing the current period's capacity, we are not limited in how high we set the inventory position after ordering. The logic of the optimal policy is such that y_t should be raised to the optimal base-stock level $\hat{y}_t$, although in fact y_t does not reflect the actual inventory position as it is possible that the order will not be delivered in its full size.

In a stationary demand and capacity setting, the base-stock levels are increased above the normal inventory level required to satisfy the expected demand. By doing so the extra inventory in the form of safety stock is kept to account for the uncertainty in future demand and supply. The uncertainty can lead to demand/supply mismatches, and correspondingly to increased inventory holding and backorder costs. In the context of our model, the dependency

Table 2: Optimal base-stock levels $\hat{y}_t(z_{t-2}, z_{t-1})$ ($L = 2$, $m = 2$, $E[D] = 5$, $CV_D = 0$, $E[Q] = 6$, $CV_Q = 0.33$)

	z_{t-1}									
z_{t-2}	0	1	2	3	4	5	6	7	8	9
0	20	20	20	20	20	20	21	22	23	24
1	20	20	20	20	20	20	21	22	23	24
2	20	20	20	20	20	20	21	22	23	24
3	20	20	20	20	20	20	21	22	23	24
4	20	20	20	20	20	21	21	22	23	24
5	20	20	20	20	21	21	22	23	**23**	24
6	21	21	21	21	21	22	22	23	24	25
7	22	22	22	22	22	23	23	24	25	25
8	23	23	23	23	23	**24**	24	25	25	26
9	24	24	24	24	24	24	25	25	26	27

of the optimal base-stock level on $\vec{z}_t$ can be intuitively attributed to the following; if we have been placing high orders (with regards to expected supply capacity available) in past periods, it is likely that a lot of the orders will not be realized in their entirety. This leads to probable replenishment shortages and demand backordering due to insufficient inventory availability. Therefore it is rational to set the optimal base-stock level higher with a goal of taking advantage of every bit of available supply capacity in the current period. By setting high targets, we aim to get the most out of the capacity, that is, we want to take advantage of periods with high supply availability, although the chances that it will actually be realized can be small. If currently, we are not facing supply shortages the tendency to use the above logic diminishes. The result is also confirmed in Table 2, where we see that the optimal base-stock level is increasing with increasing uncertain pipeline orders z_{t-1} and z_{t-2}.

4. Insights from the myopic policy

We proceed by establishing the approximate inventory policy that would capture the relationship between the uncertain pipeline orders and the target inventory position. While the optimal policy is obtained through a minimization of the multi-period cost-to-go function J_t as given in (5), the approximate policy is a solution to a single-period cost function $C_t(y_t, \vec{z}_t, z_t)$ as given in (3), thus we can refer to it as a myopic solution. The resulting structure of the myopic policy is equivalent to the structure of the optimal policy as presented in Part 2 of Theorem 2; however the orders are placed up to myopic base-stock levels $\hat{y}_t^M(\vec{z}_t)$, rather than optimal base-stock levels $\hat{y}_t(\vec{z}_t)$. Observe that $\hat{y}_t^M(\vec{z}_t)$ are also state-dependent on the uncertain pipeline orders $\vec{z}_t$, thus the myopic policy is able to account for the possible shortage in supply of the current pipeline orders. A detailed derivation of the myopic solution is provided in Lemma 2 in the Appendix.

While it would be great if myopic policy would provide a reliable estimate of the optimal costs, one can easily see that the myopic policy cannot account for future supply shortages (that is for the orders that are still to be placed in the future). This holds particularly for highly utilized system settings and in the case of high demand/supply capacity uncertainty that leads to probable demand and supply mismatches. Thus, the following study is primarily concerned with capturing the state-dependency of the optimal base-stock levels by exploring the relationship between the vector of uncertain pipeline orders and the corresponding approximate base-stock levels.

In Table 3 we present the base-stock levels for the optimal policy, myopic policy and the differences between the two. Both, the optimal and the myopic, base-stock levels were determined through the numerical analysis by minimizing the relevant multi-period and single period cost functions as mentioned above. Note that myopic policy is optimal when there is no supply capacity uncertainty (which corresponds to a base-stock level of 23). Looking at the differences between the base-stock levels we see that the myopic base-stock levels are always lower and thus can be regarded as a lower bound for the optimal base-stock levels. The differences are decreasing with increasing uncertain pipeline orders. This can be attributed to the fact that myopic policy accounts for the potential shortages in the replenishment of the pipeline orders, but fails to account for future supply unavailability. For high uncertain pipeline orders, the additional inventory to cover the supply shortage is sufficient to cover the risk of future shortages. in fact, we observe a risk pooling effect, where the base-stock level of 29 is sufficient to cover both risks.

5. Value of ASI

In this section we estimate the extent of the savings gained through incorporating ASI into the inventory system. We perform a numerical analysis to quantify the value of ASI and assess the influence of the relevant system parameters. Numerical calculations were done by solving the dynamic programming formulation given in (4).

To determine the influence of ASI parameter m, demand uncertainty, supply capacity uncertainty, and system utilization on the value of ASI, we set up the base scenario that is characterized by the following parameters: $T = 10, L = 3, \alpha = 0.99$ and $h = 1$ and

Table 3: The myopic and optimal base-stock levels, ($L=2$, $m=2$, $E[D]=5$, $CV_D=0.5$, $E[Q]=6$, $CV_Q=0.33$)

Myopic					z_{t-1}					
z_{t-2}	0	1	2	3	4	5	6	7	8	9
0	23	23	23	23	23	23	24	24	25	26
1	23	23	23	23	23	23	24	24	25	26
2	23	23	23	23	23	23	24	24	25	26
3	23	23	23	23	23	23	24	24	25	26
4	23	23	23	23	23	23	24	25	25	26
5	23	23	23	23	23	24	24	25	26	27
6	24	24	24	24	24	24	25	25	26	27
7	24	24	24	24	25	25	25	26	27	28
8	25	25	25	25	25	26	26	27	28	29
9	26	26	26	26	26	27	27	28	29	29
Optimal					z_{t-1}					
z_{t-2}	0	1	2	3	4	5	6	7	8	9
0	27	27	27	27	27	27	27	28	29	29
1	27	27	27	27	27	27	27	28	29	29
2	27	27	27	27	27	27	27	28	29	29
3	27	27	27	27	27	27	27	28	29	29
4	27	27	27	27	27	27	28	28	29	29
5	27	27	27	27	27	27	28	29	29	29
6	27	27	27	27	28	28	28	29	29	29
7	28	28	28	28	28	29	29	29	29	29
8	29	29	29	29	29	29	29	29	29	29
9	29	29	29	29	29	29	29	29	29	29
Difference					z_{t-1}					
z_{t-2}	0	1	2	3	4	5	6	7	8	9
0	4	4	4	4	4	4	3	4	4	3
1	4	4	4	4	4	4	3	4	4	3
2	4	4	4	4	4	4	3	4	4	3
3	4	4	4	4	4	4	3	4	4	3
4	4	4	4	4	4	4	4	3	4	3
5	4	4	4	4	4	3	4	4	3	2
6	3	3	3	3	4	4	3	4	3	2
7	4	4	4	4	3	4	4	3	2	1
8	4	4	4	4	4	3	3	2	1	0
9	3	3	3	3	3	2	2	1	0	0

$b = 20$. A discrete uniform distribution is used to model demand and supply capacity where the expected demand is given as $E[D] = 4$ and the expected supply capacity varies $E[Q] = \{4, 6, 8\}$, which means that the utilization of the system is $Util = \{1, 0.75, 0.5\}$. In addition we vary the coefficient of variation of demand $CV_D = \{0, 0.65\}$ and supply capacity $CV_Q = \{0, 0.33, 0.65\}$, and the ASI parameter $m = \{3, 2, 1, 0\}$, covering both the *No information* and *Full information* case.

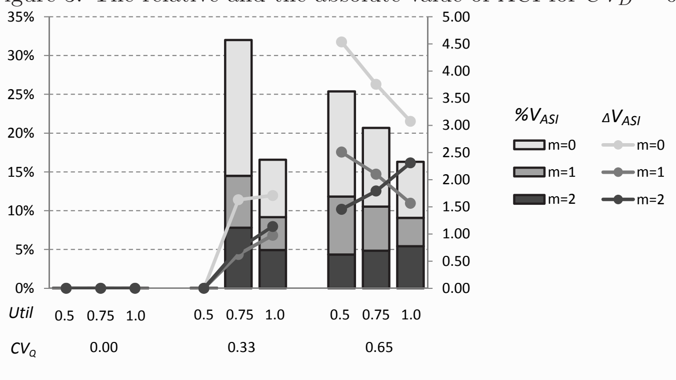
We define the relative value of ASI for $m \leq L$, $\%V_{ASI}$, as the relative difference between the optimal expected cost of managing the system in the *No information* case ($m = L$), and the system where we have obtained ASI on a number of pipeline orders ($m \leq L$):

$$\%V_{ASI}(m \leq L) = \frac{f_t^{(m=L)} - f_t^{(m\leq L)}}{f_t^{(m=L)}}. \tag{8}$$

We also define the marginal change in the value of ASI, $\triangle V_{ASI}$. With this we measure the extra benefit gained by decreasing the number of uncertain pipeline orders by obtaining ASI sooner, from m to $m-1$:

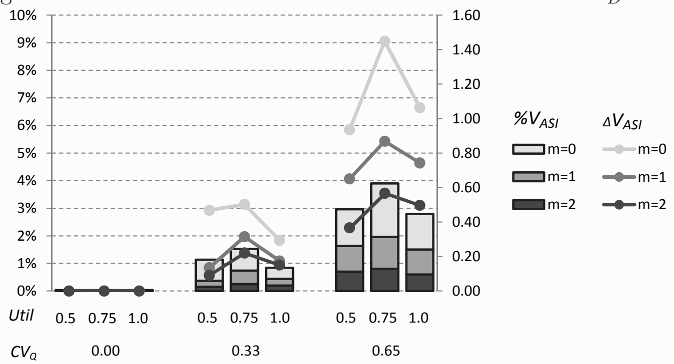
$$\triangle V_{ASI}(m-1) = f_t^{(m)} - f_t^{(m-1)}$$

Figure 3: The relative and the absolute value of ACI for $CV_D = 0$.



We present the results in Figures 3 and 4. The interplay of system parameters is relatively complex, which is exhibited in the fact that the value of ASI changes in a non-monotone manner. This holds in the case of changing the system's utilization, where the majority of the gains are made at (in our case) moderate utilizations. Increasing capacity uncertainty is where we would anticipate that the value of ASI will be increasing and majority of the

Figure 4: The relative and the absolute value of ACI for $CV_D = 0.65$.



gains would be made. This can be observed in $\triangle V_{ASI}$ for both low and high demand uncertainty scenario, while this only partially holds for $\%V_{ASI}$. The increasing demand uncertainty decreases both the relative and the absolute value of ASI. While the relative value of ASI extends over 30%, and for most of the scenarios above 10%, it drops below 4% for all scenarios under high demand uncertainty. This is expected, as the benefits will depend on to what extent the uncertainties in the system can be resolved. In the case of low demand uncertainty, ASI lowers the prevalent supply uncertainty. While in the high demand uncertainty scenario, supply uncertainty only represents a part of the total uncertainty to be resolved, and correspondingly the value of ASI is lower under this scenario. Observe also, that under the assumption of perfect ASI, the value of ASI observed represents the upper bound on the potential benefits obtained through upstream supply visibility.

The interesting observation is made when studying the influence of the ASI parameter m. Improving the ASI visibility by decreasing m from $m = L$ in the *No information* case to $m = 0$ in the *Full information*, leads to increasing marginal returns in most of the scenarios studied. As seen on the Figures 3 and 4, this holds both in the case of relative change, as well as absolute change in the optimal costs. The only outlier is the scenario with high utilization, low demand uncertainty and high capacity uncertainty in Figure 3. While we cannot conclude that the marginal returns are always increasing with the extended ASI visibility, it is clear that the majority of the benefits are gained only when we approach *Full information* case.

6. Conclusions

In this paper, we analyze a periodic review inventory system with positive lead time and stationary stochastic demand and supply capacity. As the *No information* case of our model can be considered as a positive lead time generalization of the paper by Ciarallo et al. (1994), we also extend the scope of the model by incorporating the possibility to obtain information about the available supply capacity for the pipeline orders. ASI is revealed after the order has been placed, but before it is replenished.

We show that the optimal policy is highly complex due to the extensive system's state description, where apart from the inventory position, the stream of uncertain pipeline orders has to be monitored and adapted constantly. However, despite this complexity, we show that the optimal policy is a state-dependent base-stock policy. We show that the base-stock levels should be increased to compensate for the increased replenishment uncertainty. Despite the fact that the myopic policy does not provide a good approximation for the optimal base-stock levels (and optimal costs), we show that by inclusion of the safety factor that would compensate for the future supply capacity uncertainty, the myopic policy adequately captures the risk of shortages in pipeline orders.

Numerical calculations show that the benefits obtained through ASI can be relatively big (although also highly dependent on other system parameters), however in this case ASI should be revealed for the most of the uncertain pipeline orders as we observe the increasing returns with the increasing ASI availability.

The analysis in the future could explore different alternatives to the presented ASI model. These could go into two general directions: further simplification of the system under study, or the opposite, the study of a more realistic supply chain setting. Due to the complexity of the optimal policy, obtaining the optimal parameters is still a formidable task, thus further insights could be gained by studying simplified settings (for instance constant demand, Bernoulli distributed supply capacity, etc.). For these settings explicit expressions could be obtained that would better capture the supply uncertainty structure in the system, and lead to easier determination of the base-stock levels. On the other hand, the natural extension to the single-stage inventory models is studying the multi-tier supply chains or supply networks. While these represent additional modeling challenges, one should recognize that the single-stage models provide the basic insights and act as building blocks to analyse more complex interactions in real-life supply networks. These interactions could involve capturing the uncertain supply market conditions through the use of bayesian learning to model the supply information, incentives to stimulate the information sharing in the form of supply contracts, exploring the influence of ASI on the bullwhip effect, inventory competition and allocation problems due to limited supply availability and the resulting speculative and gaming behavior of supply chain parties in response to the disclosed supply information, etc.

Appendix

Before giving the proofs, we first provide the needed notation and the definitions, which enables us to give the proofs in a concise manner. For clarity reasons, we elect to suppress the time subscripts in certain parts of the proofs. We also assume $\alpha = 1$ for the same reason.

For the m uncertain pipeline orders in $\vec{z}_t$, we know that any particular order z_i, where $i = t-m \dots t-1$ can either be delivered in full or only partially depending on the available supply capacity revealed through ASI. Based on this we define the vector $\vec{z}_t^-$, which represents the set of orders z_i that will not be delivered in full, $z_i > Q_i$. The vector $\vec{z}_t^+$ represents the set of orders z_i that will be fully replenished, $z_i \le Q_i$. Thus, $\vec{z}_t^- \cap \vec{z}_t^+ = \vec{z}_t$ and $\vec{z}_t^- \cup \vec{z}_t^+ = \varnothing$ holds. As it will be useful in some of the following derivations to include also the order z_t into the two vectors $\vec{z}_t^-$ and $\vec{z}_t^+$, we also define the extended vectors $\vec{Z}_t^-$ and $\vec{Z}_t^+$. The two corresponding supply capacity vectors are denoted as $\vec{Q}_t^-$ and $\vec{Q}_t^+$.

We denote the cumulative distribution function of the demand with $G(D_t)$, and the corresponding probability density function with $g(D_t)$, and the lead time demand counterparts with $G_t^L(D_t^L)$ and $g_t^L(D_t^L)$. The cumulative distribution function and the probability density function of supply capacity Q_t are denoted as $R_t(Q_t)$ and $r_t(Q_t)$. We assume that all the distributions are stationary.

In the following lemma, we provide the convexity results and the optimal solution to a single period cost function $C_t(y_t, \vec{z}_t, z_t)$.

Lemma 2 *Let $\hat{y}_t^M$ be the smallest minimizer of $C_t(y_t, \vec{z}_t, z_t)$ to which the optimal order $\hat{z}_t^M$ is placed, where $\hat{z}_t^M = \hat{y}_t^M - x_t$:*

1. *$C_t(y_t, \vec{z}_t, z_t)$ is convex in y_t.*

2. *$C_t(y_t, \vec{z}_t, z_t)$ is quasiconvex in z_t.*

3. *$\hat{y}_t^M(\vec{z}_t)$ is the state-dependent optimal myopic base-stock level.*

Proof: $C(y,\vec{z},z)$ is expressed in the following way:

$$C(y,\vec{z},z) = b\int_0^{\vec{Z}^-}\int_{\vec{Z}^+}^{\infty}\int_{y-\sum(\vec{Z}^- - \vec{Q}^-)}^{\infty}\left(D^L - y + \sum(\vec{Z}^- - \vec{Q}^-)\right) r(\vec{Q})\mathrm{d}\vec{Q}g^L(D^L)\mathrm{d}D^L$$
$$+h\int_0^{\vec{Z}^-}\int_{\vec{Z}^+}^{\infty}\int_0^{y-\sum(\vec{Z}^- - \vec{Q}^-)}\left(y - \sum(\vec{Z}^- - \vec{Q}^-) - D^L\right) r(\vec{Q})\mathrm{d}\vec{Q}g^L(D^L)\mathrm{d}D^L. \quad (A1)$$

To prove Part 1, we derive the first partial derivative of (A1) with respect to y, where we take into account that $\prod\left((1-R(\vec{Z}^+))R(\vec{Z}^-)\right) = 1$:

$$\frac{\partial}{\partial y}C(y,\vec{z},z) = -b + (b+h)\prod(1-R(\vec{Z}^+))\int_0^{\vec{Z}^-} G^L\left(y - \sum(\vec{Z}^- - \vec{Q}^-)\right) r(\vec{Q}^-)\mathrm{d}\vec{Q}^-, \quad (A2)$$

and the second partial derivative:

$$\frac{\partial^2}{\partial y^2}C(y,\vec{z},z) = (b+h)\prod(1-R(\vec{Z}^+))\int_0^{\vec{Z}^-} g^L\left(y - \sum(\vec{Z}^- - \vec{Q}^-)\right) r(\vec{Q}^-)\mathrm{d}\vec{Q}^-. \quad (A3)$$

Since all terms in (A3) are nonnegative, Part 1 holds. It is easy to see that the convexity also holds in x.

To show Part 2, we obtain the first two partial derivatives of $C(y,\vec{z},z)$ with respect to z:

$$\begin{aligned}\frac{\partial}{\partial z}C(y,\vec{z},z) &= (b+h)(1-R(z)) \\ &\quad \left[\prod(1-R(\vec{z}^+))\int_0^{\vec{z}^-} G^L\left(y - \sum(\vec{z}^- - \vec{Q}^-)\right) r(\vec{Q}^-)\mathrm{d}\vec{Q}^- - \frac{b}{b+h}\right], \quad (A4)\end{aligned}$$

$$\begin{aligned}\frac{\partial^2}{\partial z^2}C(y,\vec{z},z) &= -r(z)(b+h)\left[\prod(1-R(\vec{z}^+))\int_0^{\vec{z}^-} G^L\left(y - \sum(\vec{z}^- - \vec{Q}^-)\right) r(\vec{Q}^-)\mathrm{d}\vec{Q}^- - \frac{b}{b+h}\right] \\ &+ (b+h)(1-R(z))\left[\prod(1-R(\vec{z}^+))\int_0^{\vec{z}^-} g^L\left(y - \sum(\vec{z}^- - \vec{Q}^-)\right) r(\vec{Q}^-)\mathrm{d}\vec{Q}^-\right]. \quad (A5)\end{aligned}$$

Setting (A4) to 0 proves Part 3. Observe that $\hat{y}^M(\vec{z})$ only depends on $\vec{z}$, and not on z. Intuitively this makes sense, as due to the potential shortages in replenishment of any of uncertain pipeline orders $\vec{z}$ we increase $\hat{y}^M$ accordingly. However, when placing order z, it is not rational to adjust $\hat{y}^M$ to account for the potential shortage in replenishment of z. One merely has to hope that by ordering z up to $\hat{y}^M$, the available supply capacity will be sufficient.

For $z \le \hat{z}^M$, the bracketed part in the first term of (A5) is not positive, thus the first part is nonnegative as a whole. Since also the second term is always nonnegative, the function

$C(y,\vec{z},z)$ is convex on the respected interval. For $z > \hat{z}^M$ this does not hold, however we see that (A4) is nonnegative, thus $C(y,\vec{z},z)$ is nondecreasing on the respected interval, which proves Part 2. Due to this, the $C(y,\vec{z},z)$ has a quasiconvex form, which is sufficient for $\hat{y}^M$ to be its global minimizer.

Note, that one can show that $C(y,\vec{z},z)$ is quasiconvex in any of z_i, where $i = t - m \dots t-1$, in the same way as presented above. The above derivation can be considered as a generalization of the derivations for the zero lead time model presented in Ciarallo et al. (1994). We have shown that the convexity properties of the single period function also holds for the positive lead time case. □

Proof of Lemma 1: The proof is by induction on t. For period T it holds $J_T(y_T,\vec{z}_T,z_T) = C_T(y_T,\vec{z}_T,z_T)$, which by using the result of Lemma 2 proves the convexity of $J_T(y_T,\vec{z}_T,z_T)$ in y_T, and using (6) for T, also the convexity of $f_T(y_T,\vec{z}_T)$ in x_T.

Assuming that f_{t+1} is convex in x_{t+1}, we now want to show that this implies convexity of J_t in y_t and f_t in x_t. Using (5) we write J_t as:

$$\begin{aligned} J_t(y_t,\vec{z}_t,z_t) &= C_t(y_t,\vec{z}_t,z_t) \\ &+ \int_0^\infty \int_0^{z_{t-m}} f_{t+1}(y_t - (z_{t-m} - Q_{t-m}) - D_t, \vec{z}_{t+1}) r(Q_{t-m}) \mathrm{d}Q_{t-m} g(D_t) \mathrm{d}D_t \\ &+ (1 - R(z_{t-m})) \int_0^\infty f_{t+1}(y_t - D_t, \vec{z}_{t+1}) g(D_t) \mathrm{d}D_t. \end{aligned} \tag{A6}$$

By taking the second partial derivative of (A6) with respect to y_t we quickly see that the convexity of J_t in y_t is preserved due to the convexity of C_t in y_t coming from Lemma 2, while the the remaining two terms are convex due to the induction argument.

To show that this also implies convexity of f_t in x_t, we first take the first partial derivative of J_t with respect to z_t[1]:

$$\begin{aligned} \frac{\partial}{\partial z} J(y_t,\vec{z}_t,z_t) &= \frac{\partial}{\partial z} C(y_t,\vec{z}_t,z_t) \\ &+ \int_0^\infty \int_0^{z_{t-m}} f'_{t+1}(y_t - (z_{t-m} - Q_{t-m}) - D_t, \vec{z}_{t+1}) r(Q_{t-m}) \mathrm{d}Q_{t-m} g(D_t) \mathrm{d}D_t \\ &+ (1 - R(z_{t-m})) \int_0^\infty f'_{t+1}(y_t - D_t, \vec{z}_{t+1}) g(D_t) \mathrm{d}D_t, \end{aligned} \tag{A7}$$

Partially differentiating (6) with respect to x_t twice, using (A2) and taking into account

[1]We define the first derivative of a function $f_t(x)$ with respect to x as $f'_t(x)$.

the first-order optimality condition by setting (A7) to zero, yields the following:

$$\begin{aligned}\frac{\partial^2}{\partial x_t^2} f(x_t, \vec{z}_t) &= (b+h)\prod(1-R(\vec{Z}_t^{\,+}))\int_0^{\vec{Z}_t^{\,-}} g^L\left(x_t+\hat{z}_t-\sum(\vec{Z}_t^{\,-}-\vec{Q}_t^{\,-})\right) r(\vec{Q}_t^{\,-})\mathrm{d}\vec{Q}_t^{\,-} \\ &+ \int_0^{\infty}\int_0^{z_{t-m}} f''_{t+1}(x_t+\hat{z}_t-(z_{t-m}-Q_{t-m})-D_t,\vec{z}_{t+1})r(Q_{t-m})\mathrm{d}Q_{t-m}g(D_t)\mathrm{d}D_t \\ &+ (1-R(z_{t-m}))\int_0^{\infty} f''_{t+1}(x_t+\hat{z}_t-D_t,\vec{z}_{t+1})g(D_t)\mathrm{d}D_t. \qquad \text{(A8)}\end{aligned}$$

While the expression does not get simplified as in the zero lead time case, we can easily conclude that the convexity of f_t in x_t is also preserved as all the terms above are nonnegative. □

Proof of Theorem 1: The convexity results of Lemmas 1 and 2 imply the proposed optimal policy structure. □

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