

WHERE IS AUSTRIA'S ECONOMY HEADING?

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ABSTRACT: *Austria is one of the wealthiest and most stable European Union Member States. With spectral analysis this paper attempts to forecast economic indicators of the Austrian economy up to 2030 to provide a clearer picture of its future economy. The applied spectral analysis reveals hidden periodicities in the studied country's economic data which are to be associated with cyclical behaviour or recurring processes in economic time series. The 2018-2030 period forecasts of Austria's real GDP, government budget deficit or surplus in current prices, current account balance and total population respectively are all bullish, including unemployment rate doomed to expand at an annual rate of 0.58% until 2030.*

Key words: *Austria, Gross Domestic Product, forecast, spectral analysis, Burg model*

JEL classification: C53, E37

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1 INTRODUCTION

Austria's economy is a well-developed market economy with skilled labour force and high standard of living (Index Mundi, 2018). It has developed strong ties with the European Union (EU) economies such as Germany, otherwise the economic leader of the EU block. The studied country's economy witnesses a predominant service sector, a rather developed industrial sector, and a small but highly developed agricultural sector respectively. Tourism is a strong component of Austria's economy, being one of the largest natural land reserves in central Europe. Mechanical engineering, steel construction, chemicals, luxury commodities, vehicle manufacturing, and food are the most significant industries of the country. The industrial and commercial sector respectively are characterized by a high proportion of medium-sized companies. The growth of the industrial sector requires additional imports, while in the sector of raw materials and energy production, Austria has natural resources of iron ore, non-ferrous metals, important minerals and earths. In addition, the country generates its own resources of petroleum and natural gas, and what is more, is the leader of hydroelectric power in the EU, which however needs to be constantly expanded.

Austria joined the euro area in 2002. The euro area is a monetary union of 19 of the 28 European Union member states which have adopted the euro currency as their common currency and sole legal tender. Besides Austria, the other countries belonging to the euro

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area are Belgium, Finland, France, Germany, Ireland, Italy, Luxembourg, Netherlands, Portugal, Spain, Greece, Slovenia, Cyprus, Malta, Slovakia, Estonia, Latvia, and Lithuania.

In 2018, Austria had the 7th largest real GDP among the euro area countries, with lagging behind Germany, France, Italy, Spain, Netherlands, and Belgium. The correlation of Austria's real GDP with the six top performers of the euro area is illustrated in Table 1.

Table 1: *Correlation matrix between the top 7 Real Gross Domestic Products of the euro area of Chained 2010 Euros, Seasonally Adjusted, Frequency: quarterly from 1995-01-01 to 2018-01-01 (93 pieces of data)*

	Germany	France	Italy	Spain	Netherlands	Belgium	Austria	Euro area
Germany	1.00	0.96	0.54	0.87	0.95	0.97	0.97	0.95
France	0.96	1.00	0.70	0.96	0.99	0.99	1.00	0.99
Italy	0.54	0.70	1.00	0.83	0.66	0.65	0.59	0.75
Spain	0.87	0.96	0.83	1.00	0.96	0.95	0.94	0.98
Netherlands	0.95	0.99	0.66	0.96	1.00	0.99	0.99	0.99
Belgium	0.97	0.99	0.65	0.95	0.99	1.00	1.00	0.99
Austria	0.97	1.00	0.59	0.94	0.99	1.00	1.00	0.99
Euro area	0.95	0.99	0.75	0.98	0.99	0.99	0.99	1.00

Source: Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org> and Eurostat, <https://ec.europa.eu/eurostat/data/database>

Table 1 illustrates the strong relationships that Austria's economy has developed with the top six performer economies of the euro area with the exception of Italy (0.59 only). Overall, the correlation coefficient of Austria's real GDP with the euro area real GDP tops 0.99 through the 1995-2018 period (93 observations).

Based on the GDP per capita world ranking (The World Factbook, 2019), the top seven performers of the euro area rank in the following order: the Netherlands (USD 53,600 per capita 2017 estimate, 23th world ranking), Germany (USD 50,400, 27th), Austria (USD 49,900, 30th), Belgium (USD 46,600, 35th), France (USD 43,800, 41st), European Union (USD 40,900, 45th), Spain (USD 38,300, 49th), and Italy (USD 38,100, 50th).

Austria was one of the few countries in the Eurozone to emerge relatively unharmed from the 2008-2009 financial crisis (Famira-Mühlberger & Leoni, 2013) thanks to its high GDP per capita, its high employment and on the other hand relatively low unemployment. In January 2019, with 4.8% the Austrian unemployment rate was the 10th lowest rate in the euro area, with the top three lowest unemployment rate countries being the Czech Republic (2.1%), Germany (3.2%), and the Netherlands (3.6%), including the euro area recording 7.8% (Statista, 2019). After 2008, employment growth in Austria was markedly

higher than the average for the euro area and the rise in unemployment was much lower. In 2013, Austria had a higher real GDP level than before the crisis, i.e. in 2007, which was true only for about half of the euro area member countries. Over the longer term, Austria has benefited from the above-average growth attributed in part to its efficient institutions and the ability to adapt to changing conditions, and in part to the positive impact of the EU eastern enlargement, including the country's geographical location in a strong and dynamic economic region. However, compared to its neighbour Switzerland (Weyerstrass, 2015), which is not in the EU but is of similar size and geographical location, Austria has fallen behind in recent years. In fact, in 1995, Austria's GDP per capita was 14 per cent lower than that of Switzerland. By 2017, this gap had widened to 23 per cent based on the authors' computation. According to Weyerstrass, the slowdown has had both external and domestic reasons. The most important domestic demand component, i.e. private consumption, has only grown very little since 2011. Reasons for this situation have been a stubbornly high inflation rate, compared to the euro area average and to Germany, but also a rising tax burden. The Austrian income tax system is quite progressive, that is the country's nominal wage increases are to a large extent eaten up by higher taxes. In addition, the government has raised indirect taxes so as to curb the fiscal deficit. What is more, to flat its private consumption, the typical engine of economic recoveries in Austria has stuttered of late. Typically, first Austria's exports would recover, followed by companies starting to invest more. However, in the recent past and the impact still felt today, Austrian export performance has been disappointing. This has not only been caused by a lack of foreign demand but by the fact that Austrian companies have also lost ground on international markets.

This paper aims at forecasting Austria's economic indicators until 2030, using signal processing. The latter focuses on decorticating signals to capture, fathom, control, and extrapolate information nested in these signals (Rostan and Rostan, 2018a). Section 2 of the paper reviews the literature on the applied economic forecasting methods and the place of signal processing among them.

2 THEORETICAL BACKGROUND

The traditional economic forecasting methods include causal methods (regression analysis, logit, probit), time series methods (moving average, exponential smoothing, trend and seasonal decomposition, Box-Jenkins ARIMA), and qualitative methods respectively (Delphi Method, Jury of Executive Opinion, Sales Force Composite, Consumer Market Survey) (FHI, 2019). Signal processing presented in this paper to forecast Austria's economic indicators belongs to the time series methods. This type of processing is borrowed from the field of physics and focuses on the analysis, synthesis, and modification of signals. The basic assumption of this paper is that economic time series behave like signals propagating through time instead of propagating through space like physics phenomena, such as audio, video, speech, geophysical, sonar, radar, medical or musical signals (IEEE, 2019). The two reasons of using the signal processing method is firstly, the outstanding versatility of signal processing in analysing and forecasting signals, and secondly, the wave pattern

that characterized the shape of the economic indicators under examination in terms of absolute level or first difference.

The advanced economic time series forecasting methods include combination of methods such as autoregressive integrated moving average (ARIMA) and seasonal exponential smoothing (SES) techniques (Rimaitytė et al., 2017), or standard linear autoregressive (AR) models where the three most commonly used nonlinear models differ in their description of the transition between regimes (Korenok, 2011). In the threshold autoregressive (abbreviated, TAR) model, regime changes abruptly, while in the smooth threshold autoregressive (abbreviated, STAR) model, regime changes slowly. Nevertheless, in both models the regime change depends on the time index or lagged values. In the Markov-switching autoregressive (abbreviated, MAR) model, however, the regime change depends on the past values of an unobserved random variable, the state of the Markov chain, and possibly the lagged values. The proposed method of this paper is a combination of methods, the wavelet analysis which is an application of signal processing, and the Burg model which fits a p^{th} order autoregressive (AR) model to the input signal x , by minimizing (least squares) the forward and backward prediction errors while constraining the AR parameters to satisfy the Levinson-Durbin recursion.

The theories of economic cycles have been initiated by dynamic economics where economic variables are changing due to their dynamic state. Interest rates, exchange rates, volatility of asset returns, gross domestic product, government budget deficit or surplus, current account balance in current prices, levels of employment or consumer spending all propagate through time in waveforms. These waveforms represent the shape and form of the before mentioned economic signals. The modelization of economic variables based on time element, subject to uncertain, unexpected and irregular dynamics, and where fluctuations occur, has challenged econometricians and forecasters in a quest for sophisticated models able to capture and predict the evolving behaviour, frequency, rate of change, amplitude, shape and form of these economic variables through time. Many physical phenomena, such as electrical, audio or seismic signals, propagate through space in waveforms. The basic idea of this paper is to apply a model that captures dynamics in physics and use the latter in dynamic economics. The concept of dynamics derived from physics refers to a state where there is a change, as for example movement. By analysing the system of mechanics of signals, dynamics can be understood. The wavelet analysis has stirred interest for its ability to analyse changing transient physical signals (Lee and Yamamoto 1994). According to Lee and Yamamoto (1994), wavelet analysis expands functions in terms of wavelets generated in the form of translations and dilations of a fixed function called the mother wavelet, where the resulting wavelets have special scaling properties, localized in time and frequency, permitting a closer connection between the represented function and their coefficients, and ensuring a greater numerical stability in reconstruction and manipulation.

Extending the analysis to complex-behaviour economic signals, the originality of this paper is to apply the wavelet analysis to the economic variables subject to common dynamics,

such as GDP time series of the countries pertaining to the same economic zone as is the euro area. Given that in the euro area international trade and financial transactions have been much more intense during the last decades than in the past (Leon, 2016), it is expected that shocks affecting one country affect through transmission channels to some extent another country as well. Some major transmission channels have been identified, such as trade, exchange rates, final integration, including the confidence channel that may affect international business cycles (Eickmeier, 2004). Nevertheless, from a theoretical point of view, the effect of globalization on the business cycle transmission, quantified by the above channels, remains unclear. The whole issue is complicated by the fact that in an optimum currency area, such as the Eurozone where Austria is an active participant, each participating country loses its own monetary and exchange rate instruments. However, the main characteristics of the cyclical economic indicators of Austria, as are wave length, volatility and transmission mechanisms of exogenous shocks, are captured by signal processing.

The applied signal processing focuses on the analysis, synthesis, and modification of signals. In this process, wavelets mimic signals with specific properties that make them useful for signal processing, while the spectrum analysis focuses on the data analysis of these signals. From a finite record of a stationary data sequence, spectrum analysis estimates how the total power is distributed over frequency (Stoica and Moses, 2005). Such analysis may reveal the so-called *hidden periodicities* in the researched data which are to be associated with cyclic behaviour or recurring processes in the field of meteorology or astronomy for example.

The wavelet analysis is a spectrum analysis technique which uses either the Discrete Wavelet Transform (DWT) or the Continuous Wavelet Transform (CWT). Since CWT has several properties that are not tractable, such as highly redundant wavelet coefficients (Valens, 1999), an infinite number of wavelets in the wavelet transform and no analytical solutions found for most functions of the wavelet transforms, practitioners use mostly DWT. For example, to refine the wavelet-based forecasting method, Renaud et al. (2002) proposed a redundant à trous wavelet transform and a multiple resolution signal decomposition. Combining the wavelet transform and ARIMA models, Conejo et al. (2005) forecasted day-ahead electricity prices. Further, focusing on seasonalities, Schlüter and Deuschle (2010) incorporated the wavelet transform in their forecasting models with a time-varying period and intensity respectively. Merging wavelet transform with ARIMA and GARCH models, Tan et al. (2010) proposed a price forecasting method. Integrating wavelet transform to multivariate adaptive regression splines (MARS) and to vector regression (SVR called Wavelet-MARS-SVR), Kao et al. (2013) addressed the problem of a wavelet sub-series selection and this way improved the forecast accuracy. Mixing wavelet and neural network models, Ortega and Khashanah (2013) forecasted stock returns from high-frequency financial data. Capturing the cyclicity of metal prices, Kriechbaumer et al. (2014) implemented a wavelet-autoregressive integrated moving average (ARIMA) model to forecast monthly prices of metals. In order to forecast the exchange rate movement, He et al. (2014) proposed an entropy optimized wavelet-based forecasting

model. Rostan et al. (2015) appraised the financial sustainability of the Spanish pension system by means of spectrum analysis. To separate short-run noise from long-run trends, Berger (2016) transformed a financial return series into its frequency and time domain via wavelet decomposition, and assessed the relevance of each frequency to the value-at-risk (VaR) forecast. Spectrum analysis was also applied to yield curve forecasting with a robust outcome when benchmarked to the Diebold and Li (2006) model (Rostan et al., 2017). Using signal processing and the multiscale principal component analysis, Rostan and Rostan (2017) forecasted the European and Asian populations with distinctive outcomes compared with the population projections of the United Nations. Finally, Rostan and Rostan illustrated the versatility of the wavelet analysis firstly, to the forecast of financial times series (2018a), secondly, to the forecast of Spanish (2018b) and Greek economies (2018c), and thirdly, to assess the Saudi pension system sustainability (2018d).

The paper continues with Section 3 in which the methodology is presented, followed by Section 4 which gathers the results, and section 5 which concludes the paper.

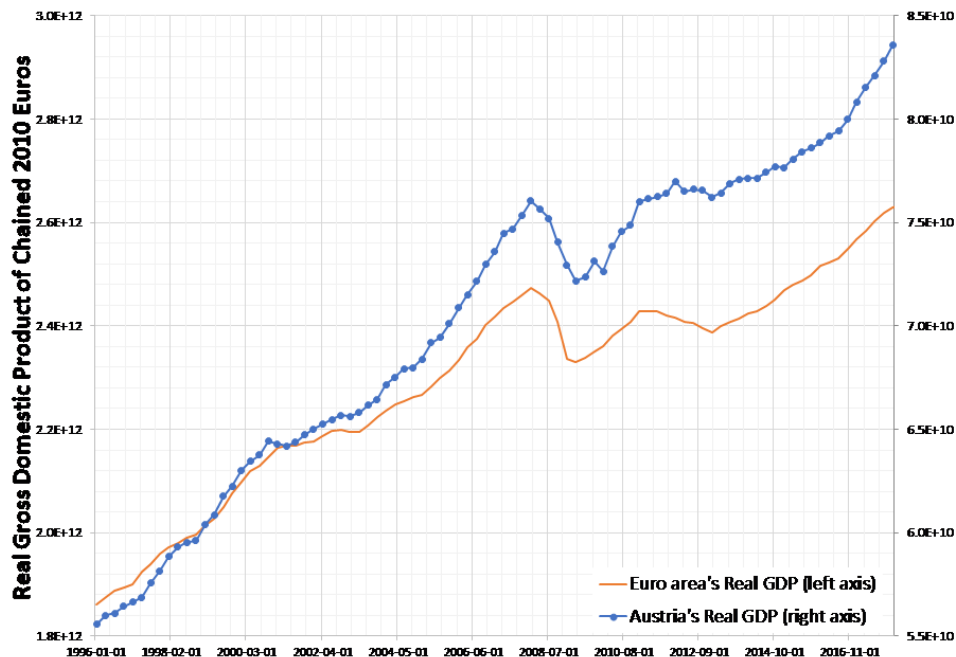
3 METHODOLOGY

The objective of the paper is to present an application of spectral analysis to the forecasts of Austria's major economic indicators to get a sense of where Austria's economy is heading by 2030. The methodology, improved with a denoising and compression step since applied to financial time series (Rostan and Rostan, 2018a), requires four steps illustrated with Austria's real Gross Domestic Product (GDP). The four steps to forecast the other economic indicators, that is the government budget deficit or surplus in current prices, current account balance, total population and unemployment rate, are identical to the four steps illustrated with GDP. Figure 1 illustrates quarterly data of Austria's real GDP of Chained 2010 Euros, seasonally adjusted, with quarterly data from 1996-01-01 to 2018-01-01 (89 pieces of data/information).

3.1 Step 1: The denoising and compression of the first-order difference of Austria's GDP time series

We compute the first-order difference of Austria's real GDP time series to transform non-stationary series into stationary series. We apply the Augmented Dickey-Fuller test to the time series before and after differentiation, where before differentiation, the time series are non-stationary, i.e. the existence of a unit root, and after differentiation, the time series is stationary, i.e. rejection of the existence of a unit root). This transformation is applied because the wavelet analysis presents a more accurate forecasting ability with stationary time series than it does with non-stationary time series. For demonstration please refer to Rostan and Rostan (2018a).

Figure 1: *Real Gross Domestic Product for Austria of Chained 2010 Euros, Seasonally Adjusted, Frequency: quarterly from 1996-01-01 to 2018-01-01 (89 data) compare to euro area real GDP*



Source: Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org> and Eurostat, <https://ec.europa.eu/eurostat/data/database>

Applying a one-dimensional denoising and compression-oriented function using wavelets, the series are denoised. The function is borrowed from Matlab (Misiti et al., 2015) and is called 'wdencmp'. The underlying model for the noisy signal is of the form:

$$s(n) = f(n) + \sigma e(n) \quad (1)$$

where time n is equally spaced, $e(n)$ is a Gaussian white noise $N(0,1)$, and the noise level σ is the standard deviation of $s(n)$ and is supposed to be equal to 1. The denoising objective is to suppress the noise part of the signal s and to recover f . The denoising procedure involves the following three steps:

- 1) Decomposition with the wavelet *sym4* and a level-2 decomposition. *Sym4* is a symlets wavelet of order 4, used as the mother wavelet for both decomposition and reconstruction. It is a nearly symmetrical wavelet, belonging to the family of *Symlets*

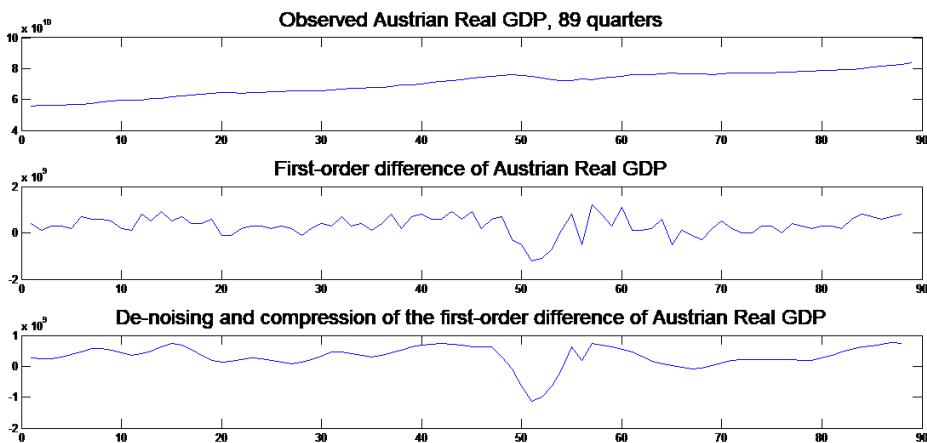
proposed by Daubechies (1994). We compute the wavelet decomposition of the signal s at level 2.

- 2) Detail coefficients thresholding. For each level from 1 to 2, we select a threshold and apply soft thresholding to the detail coefficients.
- 3) Reconstruction. We compute a wavelet reconstruction based on the original approximation coefficients of level 2 and the modified detail coefficients of levels from 1 to 2.

After denoising, the compression procedure contains three steps: 1) Decomposition. 2) Detail coefficient thresholding. For each level from 1 to 2, a threshold is selected and hard thresholding is applied to the detail coefficients. 3) Reconstruction. The difference with the denoising procedure is found in step 2. Compression is based on the concept that the regular signal component can be accurately approximated, using a small number of approximation coefficients (at a suitably selected level) and some of the detail coefficients.

Figure 2 illustrates Austria's real GDP (89 pieces of data) before differentiation (top figure), after differentiation (middle) and after denoising and compression (bottom).

Figure 2: *Observed Austria's Real GDP from 1996-01-01 to 2018-01-01 (89 pieces of data, top), First-order difference of Austria's Real GDP (middle), Denoising and Compression of the first-order difference of Austria's Real GDP (bottom)*

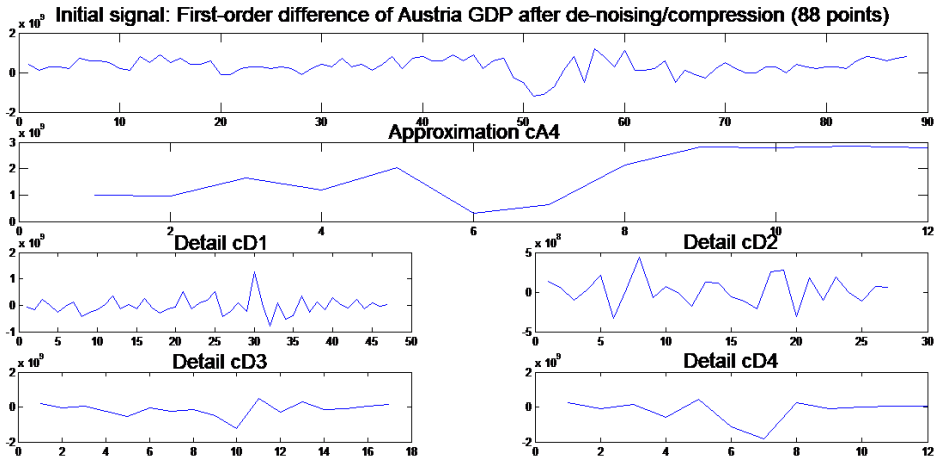


Source: Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org> and Eurostat, <https://ec.europa.eu/eurostat/data/database>

3.2 Step 2: Wavelet Decomposition

We decompose the signal after being differentiated, denoised and compressed. The signal, i.e. the 89-quarter time series of Austria's GDP transformed at step 1, is decomposed into decomposed signals cA s named approximations, and cD s named details. The Discrete Wavelet Transform is a kind of decomposition scheme evaluated by passing the signal through lowpass and highpass filters (Corinthios, 2009), dividing the signal into a lower frequency band and an upper band. Each band is subsequently divided into a second level lower and upper bands. The process is repeated, taking the form of a binary, or a "dyadic" tree. The lower band is referred to as the approximation cA and the upper band as the detail cD . The two sequences cA and cD are downsampled. The downsampling is costly in terms of data, in other words, with a multilevel decomposition, at each one-level of decomposition the sample size is reduced by half. In fact, it is reduced by slightly more than half the length of the original signal, since the filtering process is implemented by convolving the signal with a filter. The convolution "smears" the signal, introducing several extra samples into the result. Therefore, the decomposition can proceed only until the individual details consist of a single sample. Thus, the number of levels of decomposition will be limited by the initial number of the data of the signal. The level of the decomposition of the signal is left to the appreciation of the user. In this paper, we apply a 4th-level decomposition. The choice of this level is explained at the end of the Methodology section. Figure 3 illustrates the 4th-level decomposition of the transformed Austria's real GDP, after differentiation and denoising/compression, with 88 points. It is also observed in Figure 3 that details cD s are small and look like high-frequency noise, whereas the approximation $cA4$ contains much less noise than does the initial signal. In addition, the higher the level of decomposition, the lower the noise generated by details. For a better understanding of signal decomposition using discrete wavelet transform, refer to the methodology section of Rostan and Rostan (2018a).

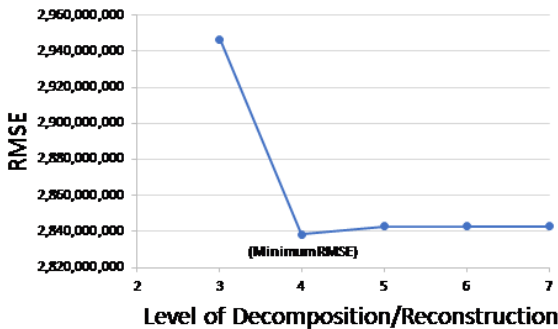
Figure 3: 4th-level decomposition of the transformed Austria's Real GDP of Chained 2010 Euros, Seasonally Adjusted (after differentiation and denoising/compression) using one-dimensional discrete wavelet analysis



Source: Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org> and Eurostat, <https://ec.europa.eu/eurostat/data/database>

To identify the optimal level of decomposition/reconstruction of our forecasting model, we make the level, varying from 1 to 7, where levels 1 and 2 return an error message. Figure 4 illustrates the RMSE computed on the last 52 in-sample quarters of our database (forecasts versus observed data) from 2005-04-01 to 2018-01-01 (52 quarters) of Austria's real GDP.

Figure 4: RMSE versus level of decomposition/reconstruction



At level-4 decomposition/reconstruction, the RMSE reaches a minimum (2.83E+09). To make the forecasts consistent between economic indicators, we apply the level-4 decomposition/reconstruction to all economic indicators, that is the government budget deficit or surplus, current account balance in current prices, unemployment rate, and total population, presented in the Results section.

3.3 Step 3: Burg extension of approximations and details

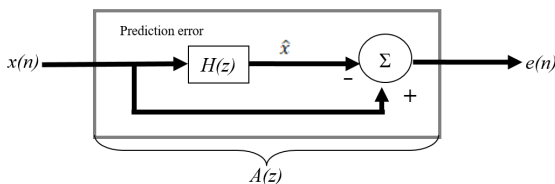
The Burg extension is applied to cA and cD , and to run the Burg extension, an autoregressive p th order from historical data is used. In this paper, we choose a p^{th} order equal to the longest available order when forecasting. For instance, in 2018-01-01, when forecasting Austria's real GDP for the subsequent quarters, the longest p^{th} order available is 84 out of the 89 collected pieces of data. Given x to the decomposed signal which is cA or cD , we generate a vector a of the all-pole filter coefficients that model an input data sequence using the Levinson-Durbin algorithm (Levinson 1946; Durbin 1960). We use the Burg (1975) model to fit a p th order autoregressive (AR) model to the input signal x , by minimizing (least squares) the forward and backward prediction errors and constraining the AR parameters to satisfy the Levinson-Durbin recursion. x is assumed to be the output of an AR system driven by white noise.

Vector a contains the normalized estimate of the AR system parameters, $A(z)$, in descending powers of z :

$$H(z) = \frac{\sqrt{e}}{A(z)} = \frac{\sqrt{e}}{1 + a_2 z^{-1} + \dots + a_{(p+1)} z^{-p}} \quad (2)$$

Since the method characterizes the input data using an all-pole model, the correct choice of the model order p is important. In Figure 5, the prediction error $e(n)$ can be viewed as the output of the prediction error filter $A(z)$, where $H(z)$ is the optimal linear predictor, $x(n)$ is the input signal, and $\hat{x}(n)$ is the predicted signal.

Figure 5: Prediction error filter to run the Burg extension



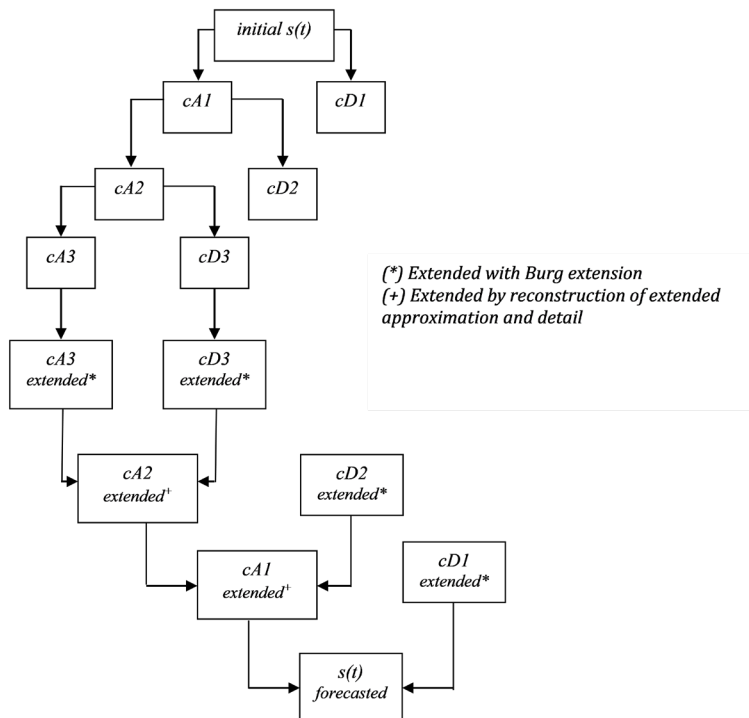
Source: Rostan and Rostan (2018a)

In the last step, the Infinite Impulse Response (IIR) filter extrapolates the index values for each forecast horizon. The IIR filters are digital filters with infinite impulse response, where unlike a finite impulse response (FIR) filter, an IIR filter provides a feedback (a recursive part of the filter) which is why it is also known as a recursive digital filter.

3.4 Step 4: Wavelet Reconstruction

After the Burg extension, the forecasted signals are recomposed, using the methodology illustrated in Figure 7. In our paper, we apply to the economic data the 4th-level decomposition/reconstruction, as explained at the end of the Methodology section. After reconstruction, the time series of the first-order difference of Austria's real GDP are transformed into Austria's real GDP absolute value. For simplification, Figure 6 illustrates a 3rd-level decomposition/reconstruction diagram.

Figure 6: Diagram of a 3rd-level wavelet decomposition/reconstruction tree to forecast the initial signal $s(t)$

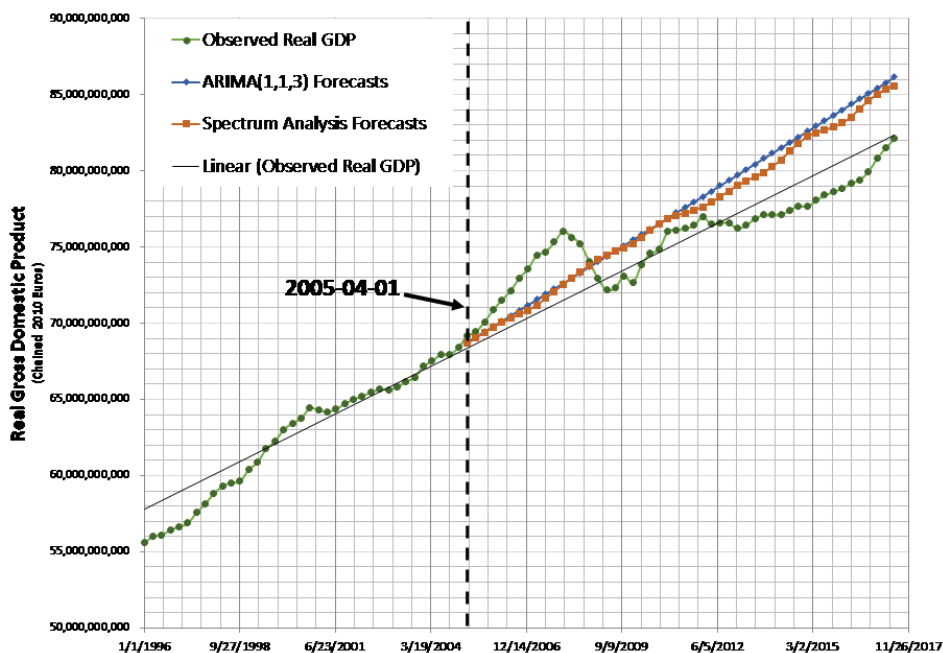


Source: own elaboration

3.5 Assessing the forecasting ability of spectrum analysis (SA)

An additional exercise within the research is to assess the forecasting ability of SA, which is done by measuring the forecasting error over the last 52 in-sample quarters of Austria's real GDP time series from 2005-04-01 to 2018-01-01. We benchmark the SA to the ARIMA(1,1,1) forecasting model (Box and Jenkins, 1976; Baillie and Bollerslev, 1992; Box et al., 1994), applied to the absolute level of GDP, i.e. no denoising and no decomposition of the time series. In addition, the Root Mean Error Square criteria (forecasts versus historical data) are applied to compute the error of forecasting. The SA beats the ARIMA(1,1,1) model with the RMSE of 2.83E+09 versus 3.21.E+09 with ARIMA over 52 quarters. The reason for using the 52 in-sample quarters data is to match our forecasting period which extends over 52 quarters from 2018-04-01 until 2031-01-01. Figure 7 illustrates the real GDP in-sample forecasts with the two models.

Figure 7: Austria's Real GDP forecasts from 2005-04-01 to 2018-01-01 (52 quarters), ARIMA(1,1,1) versus Spectrum Analysis (4th level of decomposition/reconstruction, p th order = 33)

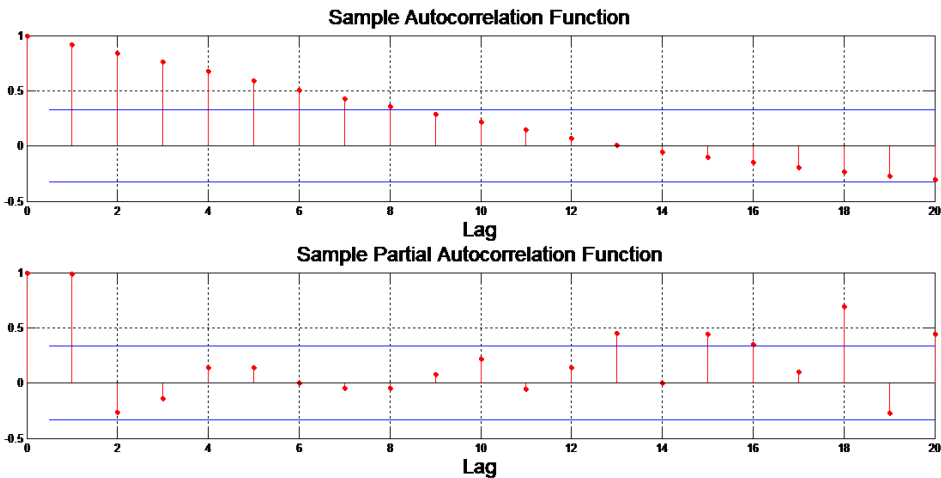


Source: Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org> and Eurostat, <https://ec.europa.eu/eurostat/data/database>

We choose the ARIMA(1,1,1) model since it best fits the data of the Austria's real GDP time series. We identify the ARMA lags $p = 1$ and $q = 3$ with the Bayesian information criterion (BIC) to the real GDP time series (37 data). For this purpose, we estimate several models with different p and q values. For each estimated model, we compute the loglikelihood objective function value. Then, we input the loglikelihood value to compute the BIC measure of fit which penalizes for complexity. The methodology involving the ARIMA model is implemented in Matlab using the econometrics toolbox.

In Figure 8, we plot the sample autocorrelation function and the partial autocorrelation function respectively of Austria's real GDP time series, illustrated in Figure 1 from 1996-01-01 to 2005-01-01 (37 pieces of data). The sample ACF decays slowly, which is consistent with the ARMA model. We obtain a rough idea of the ARMA lags by looking at the PACF. It appears that not more than one AR or MA terms are required.

Figure 8: *sample autocorrelation function (ACF) and partial autocorrelation function (PACF) of the historical Austria's Real GDP time series from 1996-01-01 to 2005-04-01 (37 pieces of data)*



Source: Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org> and Eurostat, <https://ec.europa.eu/eurostat/data/database>

To formally identify the ARMA lags, several models are fitted with different lag choices, making the degree of differencing, i.e. the "I" of ARIMA, varying from 0 to 3. We fit all combinations of ARMA(p,q) for $p = 1, \dots, 3$ and $q = 1, \dots, 3$, that is a total of 9 models per degree of differencing, when possible. The loglikelihood objective function and the number of coefficients are stored for each fitted model. We then calculate the BIC for each fitted model, by means of which the following four output BIC matrices are obtained, i.e. for no differencing, the first, second, and third order differencing respectively. In the four

output BIC matrices presented below, the rows correspond to the AR degree (p) and the columns to the MA degree (q). The optimal value in the BIC matrices is the smallest BIC value, where the smallest value ($1.0e+03*1.5429$) is obtained with the first order differencing:

		q		
$1.0e+03 *$	1.5429	1.5522	1.5570	
p	1.5582	1.5524	1.5701	
	1.5565	1.5628	1.5628	

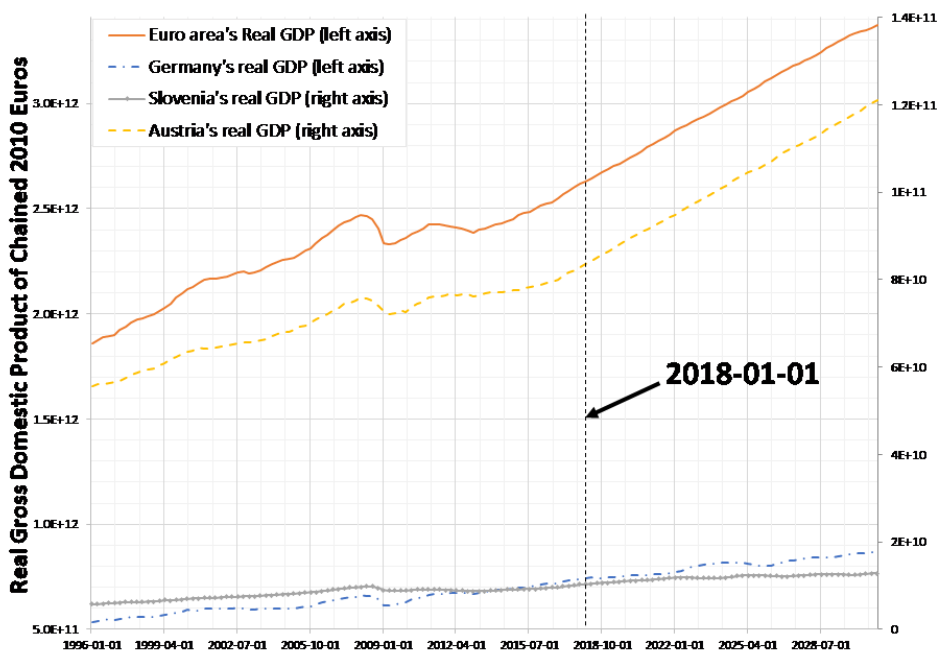
Selected model: ARIMA(p,d,q) = ARIMA(1,1,1).

4 RESULTS

The objective of the paper is to illustrate an application of the wavelet analysis to the forecast of the Austria's real GDP. In Figure 9, we illustrate 54-quarter forecasts from 2018-04-01 to 2031-01-01. The data on the left-hand side up to the vertical dot line represent Austria's real GDP quarterly data observed from 1996-01-01 to 2018-01-01 (89 pieces of data). For the decomposition/reconstruction part of the wavelet analysis, we use the 4th level, as mentioned in the Methodology section of the paper.

4.1 Forecasting Austria's Real GDP of Chained 2010 Euros, Seasonally Adjusted

Figure 9: Observed (1996-01-01 to 2018-01-01) and Forecasted (2018-04-01 to 2031-01-01) Austria's Real GDP of Chained 2010 Euros, Seasonally Adjusted, Frequency: quarterly; on the right-hand side of the vertical dot line, 52-quarter forecasts with spectrum analysis (4th-level decomposition/reconstruction, p th order = 84)



Source of historical data: Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org>, and Eurostat, <https://ec.europa.eu/eurostat/data/database>.

The sharp increase of Austria's GDP between 2018 and 2030 is benchmarked to the euro area, Germany, and Slovenia respectively. From Figure 9, the spectral analysis forecasts an expansion of Austria's, Germany's, Slovenia's, but also the euro area's economies with their respective real GDPs increasing steadily and reaching historical highs on 2031-01-01, 120,989,880,000 of Chained 2010 Euros in Austria, 3,375,212,900,000 in the euro area, 866,136,980,000 of Chained 2010 Euros in Germany, and 12,830,498,000 of Chained 2010 Euros in Slovenia. These forecasts represent an annual growth rate of +2.89% between 2018 and 2030 (13 years) of Austria's real GDP, beating the estimated annual growth rates in the euro area (+1.94%), in Germany (+1.19%), and in Slovenia (+1.68%). These estimates are benchmarked to the 2018-2030 OECD (2019) annual growth rates projections expected to be +1.48% in Austria, +1.38% in the euro area (EA16), +1.21% in Germany, and +1.26%

in Slovenia. The OECD projections are less optimistic than the spectral projections for Austria, the euro area and Slovenia respectively, but are almost equal for Germany.

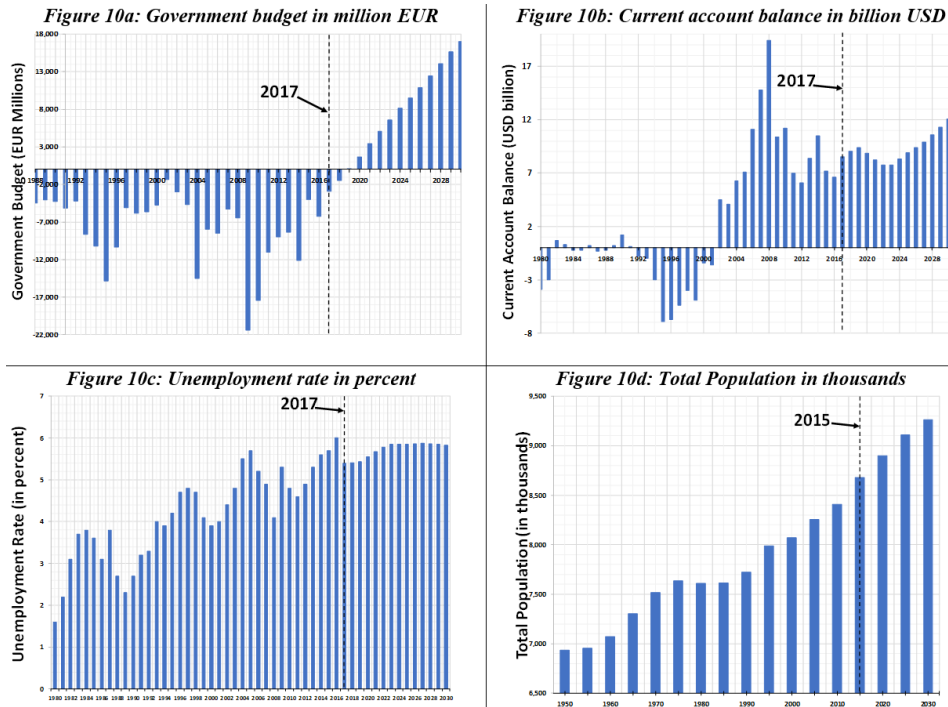
According to the Austrian Ministry of Finance (2018), in a sensitivity scenarios analysis and an optimistic outlook, the G20-countries will implement their growth strategies, the Western Balkan countries will pursue a clear EU-accession strategy and Austria will gain market shares in global trade and tourism, and private investment will accelerate.

With spectral forecasts, Austria's real GDP projection pictures a positive growth of the Austrian economy, however, additional economic indicators should be forecasted to confirm the trend. The 4-step methodology is applied to the following economic indicators:

- Government budget deficit or surplus (revenues minus expenses expressed in absolute level and in percentage of GDP).
- Current account balance in current prices.
- Unemployment rate.
- Total population.

4.2 Forecasting Austria's Government budget deficit or surplus in million euros (Revenues minus Expenses), Current account balance in billion United States dollars, Unemployment rate in percent and Total Population in thousands

Figure 10:



- **Government budget deficit or surplus in million euros (Revenues minus Expenses):** Observed (1988 to 2017) and Forecasted (2018 to 2030) Austria's government budget in million euros. Forecasts with spectral analysis (4th-level decomposition/reconstruction, p^{th} order = 27). Source of historical data: <https://countryeconomy.com/deficit/austria>.
- **Current account balance in billion United States dollars:** Observed (1980 to 2017) and Forecasted (2018 to 2030) Austria's current account balance in billion United States dollars. Forecasts with spectral analysis (4th-level decomposition/reconstruction, p^{th} order = 34). Source of historical data: <https://knoema.com/atlas/Austria/Current-account-balance>.
- **Unemployment rate:** Observed (1980 to 2017) and Forecasted (2018 to 2030) Austria's unemployment rate. Forecasts with spectral analysis (4th-level decomposition/reconstruction, p^{th} order = 34). Source of historical data: <https://knoema.com/atlas/Austria/Unemployment-rate>.
- **Total Population:** Observed (1950 to 2015) and Forecasted (2020 to 2030) Austria's total population. Forecasts with spectral analysis (4th-level decomposition/reconstruction, p^{th} order = 12). Source of historical data: United Nations, Department of Economic and Social Affairs, Population Division (2017). World Population Prospects: The 2017 Revision, DVD Edition.

Source: Federal Reserve Bank of St. Louis, <https://fred.stlouisfed.org> and Eurostat, <https://ec.europa.eu/eurostat/data/database>

The applied spectrum analysis is well-designed to forecast trends embedded in the historical data of Austria, especially the reverting trend observed with the government budget deficit starting in 2010 in Figure 10a. Figure 10a illustrates a positive trend of the government budget deficit after 2009 that should reassure the Austrian policy makers with a budget that will become a surplus of EUR +17,037 million by 2030, from EUR -2,925 million in 2017. Over the 2018-2030 period, the government budget surplus will average EUR +7,934 million according to the spectral analysis forecasts, which is well above the historical average of EUR -7,716 million. This optimistic spectral forecast has to be contrasted with the qualitative analysis of Srdoc (2017). According to Srdoc, Austria's government budget deficit compares well with other euro area countries, but the country has been exposed to numerous external risks, such as unpredictable weak demand for its exported products. Austrian banks have been exposed to Central and Eastern Europe, and the Hypo Alpe Adria bank collapse is an example of how Austrian banks may be impacted. Nevertheless, in 2009, the bank of Hypo Alpe Adria was nationalised by the Austrian government. The Carinthia state holding and Grazer Wechselseitige Versicherung sold their stakes to the Austrian government for EUR 1 each. To avoid bankruptcy, the Austrian taxpayers had to cover a loss between EUR 13 billion and EUR 19 billion of outstanding loans. In addition, the country has been exposed to political and economic uncertainties caused by the European sovereign debt crisis, the current refugee crisis, and the ongoing clashes between Russia and Ukraine (Bonenberger, 2017). According to the Austrian Ministry of Finance (2018), in an optimistic sensitivity analysis of the government budget where the world economy lacked economic tensions while existing political tensions relaxed, the public debt ratio would markedly decline below 60% of GDP already by 2021, and a significant leeway for further tax reductions would emerge for public households. The flip side of the sensitivity analysis is a pessimistic scenario where economic tensions will work themselves through the world economy and Europe, for instance through the ongoing US-China trade war where Europe could be the big loser. Political tensions will build up thanks to for instance an unorderedly Brexit whose probability of occurrence has jumped with the election of Boris Johnson, a Brexit hardliner, as Prime Minister of the United Kingdom in July 2019. This political turmoil is expected to reduce world trade and push prices of raw materials up. Corrections in asset markets will reduce consumer confidence and enterprises will hold back their investments. Nevertheless, despite the predicted weak demand, according to the analysis, the public debt ratio would decline rather modestly and public deficits would stay below -2%.

Current account balance is the sum of net exports of goods, services, net income, and net current transfers. Figure 10b illustrates the observed and forecasted Austrian current account balance. In the wave of the 2008-2009 crisis, the Austrian current account balance was badly hurt, as after reaching an all-time high of USD 19.1 billion in 2008, it plunged by more than 50% on average in the subsequent years until 2017. Thereafter, the Austrian current account balance projection is optimistic, with a transitional rally until 2019 that will lose steam up to 2023, then reverse steadily during the 2024-2030 period to reach USD 12.1 billion by 2030.

The OECD harmonized unemployment rate provides the number of unemployed persons as a percentage of the labour force, represented by the total number of people employed plus unemployed. As illustrated with Figure 10c, the historical trend of Austria's unemployment rate is positive and such will be the forecasted rate. In 2017, Austria's unemployment rate dropped by 10% to 5.4%, from 6% in 2016. Austria's 5.4% unemployment rate, while low compared to the other euro area members, flirts with its highest levels since the end of World War II, driven by an increased number of refugees and European migrants entering the labour market. For the subsequent years following the 2017 drop, the unemployment rate projection is unfortunately expected to regain momentum by 2027, topping 5.88%, then steadily decrease, thereafter reaching 5.82% in 2030, which represents an annual growth rate of 0.58% compared with 2017.

Figure 10d illustrates the steady growth of the Austrian population until 2030 to reach 9,260,707 people in 2030, which represents an annual growth of 0.43% over the 2015-2030 period. In 2016, the GDP-composition estimates by end use were represented by household consumption (52.6%), government consumption (20.1%), investment in fixed capital (23%), investment in inventories (0.8%), exports of goods and services (52.1%), and imports of goods and services (-48.6%, CIA World Factbook 2017). Nevertheless, one obvious driver of Austria's real GDP growth in the upcoming years - spectrum analysis forecasts a 2.89% annual growth - will be the Austrian population expansion since household consumption represents more than 50% of Austria's GDP.

5 DISCUSSION AND CONCLUSION

Spectral analysis (SA) is applied to the forecasts of major economic indicators of the Austrian economy up to 2030 to provide a clearer picture of the country's future economy. SA reveals hidden periodicities in data which are to be associated with the cyclical behaviour or recurring processes in economic time series. SA aims at decorticating economic data by unveiling simplified time series after decomposition, extrapolating information nested in these simplified series and rebuilding the forecasted time series. The context of the Austrian economy is pretty optimistic, as economic growth has been relatively strong in recent years in terms of real GDP, approaching 1.55% in 2015, rising to 2.50% in 2016, and jumping to 3.37% in 2017. This growth acceleration has been captured by spectral analysis, projecting a 2.89% annual growth for Austrian real GDP until 2030. Additional indicators of Austria's economy have been forecasted and all indicators, except the unemployment rate, converge to the fact that Austria's economic growth will 'lift all boats', in other words, Austria's government budget will become a surplus to EUR +17,037 million by , from -EUR 2,925 million in 2017. According to the SA forecasts, over the 2018-2030 period, the government budget surplus will average EUR +7,934 million, which is well above the historical average of EUR -7,716 million. In addition, with the applied AS the Austrian current account balance projection is bullish, experiencing a transitional rally until 2019 that will lose steam until 2023, then reverse steadily during the 2024-2030 period to reach USD 12.1 billion by 2030. The Austrian population will expand over the 2015-2030 period at an annual growth rate of 0.43%. What is more, one driver of Austria's real GDP growth

in the coming years will be the Austrian population growth since household consumption represents more than 50% of Austria's GDP. However, unemployment rate will be the larger, as with 5.4% unemployment rate in 2017, it flirts with its highest levels since the end of World War II, driven by an increased number of refugees and European migrants entering the labour market. The unemployment rate projection will unfortunately regain momentum reaching 5.82% in 2030, which represents an annual growth rate of 0.58% compared with 2017. A wealthy country as Austria is with a healthy and growing economy will continue to attract tourists, migrants and EU workers. The increasing supply of workers will unavoidably pressure up the unemployment rate. The Kurz government elected in December 2017 had made immigration control a top priority, planning to reverse the unemployment rate uptrend (Schumacher, 2017). However, Sebastian Kurz, a conservative who formed a coalition between his People's Party (ÖVP) and the far-right Freedom Party (FPÖ), was eventually ousted by Austrian lawmakers in a no-confidence vote in May 2019, following a bribery scandal involving the leader of FPÖ. Austria's president appointed an interim government led by Vice-Chancellor Hartwig Löger (BBC, 2019), and it points to Austria entering a new period of political uncertainty that should mitigate the positive outlook of economic indicators forecasted with the spectral analysis.

In conclusion, Austria's economy has accelerated over the past two years (2.50% in 2016 and 3.37% in 2017), surpassing the euro area (2.05% and 2.42% respectively). This trend has been captured by applying spectral analysis over the next 13 years between 2018 and 2030 when Austria's real GDP annual growth rate should reach 2.89% versus 1.94% for the euro area. With a predominant service sector, a rather developed industrial sector, a small, but highly developed agricultural sector, and a strong tourism sector respectively, Austria's economy should outperform the economies of most of its partners of the euro area, taking for examples the biggest and one of the smallest economies, Germany (+1.19%) and Slovenia (+1.68%). The 2018-2030 OECD annual growth rates projections are expected to be +1.48% in Austria, +1.38% in the euro area (EA16), +1.21% in Germany, and +1.26% in Slovenia. The OECD projections are less optimistic than the spectral projections for Austria, the euro area, and Slovenia respectively, but are almost equal for Germany. In addition, the 2018-2030 period forecasts of Austria's government budget deficit or surplus in current prices, current account balance and total population are all bullish, including unemployment rate doomed to expand at an annual rate of 0.58% between 2018 and 2030. However, these spectral projections should be mitigated by a negative political outlook, following three recent events that should have a crucial impact on Austria's economy in the future, namely 1) Austria's government collapse in May 2019, 2) the Brexit deal that is to be negotiated by the new Prime Minister of the United Kingdom, Boris Johnson, a Brexit hardliner, elected in July 2019, and 3) the ongoing US-China trade war, started in January 2018 by the United States president Trump, where Europe could be the biggest loser. Many European companies will indeed suffer because they both produce and sell goods in the two largest economies in the world, the US and China. For example, tariffs that China imposed on US-made autos hit German carmaker BMW since the latter produces cars in the US and exports them to China (South China Morning Post, 2018).

Additional studies, focusing on the main economic partners of Austria and incorporating in the forecasting model the comovements of their economies with Austria's economy using for instance the Multiscale principal component analysis, should refine our findings.

6 COMPLIANCE WITH ETHICAL STANDARDS

Disclosure of potential conflict of interest: The authors declare that they have no conflict of interest.

Ethical approval: This article does not contain any studies with human participants or animals performed by any of the authors.

Informed consent: Informed consent was not necessary for this study.

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