ADOPTION OF PROJECTED MORTALITY TABLE FOR THE SLOVENIAN MARKET USING THE POISSON LOG-BILINEAR MODEL TO TEST THE MINIMUM STANDARD FOR VALUING LIFE ANNUITIES

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ABSTRACT: *For the best estimate of life annuity provisions, the longevity risk of the insured population must be estimated. In this article, we present an application of the Poisson log-bilinear model to construct life annuity tables for the Slovenian market. As data on the selection effect of annuity owners are not available for the Slovenian market, we have used selection statistics from UK data. We then compare those tables with the German annuity tables DAV 1994 R, which are the current minimum standard for valuing annuity-related liabilities in Slovenia. It is shown that current minimum standard underestimate longevity risk of the insured population in Slovenia by 2–4%.*

Keywords: *Solvency II, valuation of insurance liabilities, Lee-Carter, mortality projections, life expectancy*
JEL classification: G17, G23, J11

1 INTRODUCTION

Solvency II has proposed major changes to the valuation of insurance technical provisions and has had a considerable impact on reserving processes. The Solvency II framework requires a consistent market approach to the valuation of insurance assets and liabilities. In such an approach, both assets and liabilities should be valued at the amount for which they could be transferred, or settled, between knowledgeable and willing parties (for details, see the Quantitative Impact Study 5 [QIS5]). This is a relatively new con-

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cept for insurance companies, as currently, according to existing insurance legislation, technical provisions are valued on a book-value basis (e.g., Medved, 2000).

Technical provisions represent the major part of the liability side of insurance companies' balance sheet. Insurance companies are required to establish technical provisions for all future obligations arising from insurance contracts. There are three main categories of technical provisions in the insurance industry: non-life-insurance obligations, life insurance obligations, and health insurance obligations.

According to Solvency II, the value of technical provisions is equal to the sum of a best estimate and a risk margin. The best estimate is calculated as an expected value of all future cash-out and cash-in flows, taking into account the time value of money. Cash-flow projections should reflect a realistic expectation of future demographic, legal, medical, technological, social, and economic developments. The risk margin is a buffer above the best estimate of discounted cash flows, and it protects against worse-than-expected scenarios. The risk margin covers model risks, parameter risk, and structural uncertainty. For more details on how to calculate the best estimate and risk margin, see the QIS5 technical specification.

Valuing technical provisions of life annuities depends mainly on projected demographic trends. A life annuity is a specific insurance contract in which one party (an insurance company), in exchange for payment of a premium, guarantees a series of payments until the death of the other party (the insured person). The projection of future mortality improvements has significant effects on premium calculation and reserving for life annuities (see Olivieri 2001). As such, annuities are associated with longevity risk, in that decreasing mortality rates of the insured population lead to an increase in the number of annuity payments. This article addresses the stochastic projections of future demographic trends in Slovenia as a key parameter of best-estimate valuations of Slovenian annuities.

There are no official projected mortality tables for the Slovenian population. To value life annuities, insurance companies in Slovenia must use annuity tables that are based on the mortality profile of populations in foreign countries. The Slovenian Insurance Supervision Agency has set the German annuity tables DAV 1994 R as the minimum standard. This means that insurance companies have to value their liabilities using DAV 1994 R annuity tables; however, they can use other tables, as long as those tables produce higher technical provisions than the DAV 1994 R. The result, though, is that the industry standard is to use the DAV 1994 tables for premium calculation and reserving, and in turn, mortality statistics from 1994 on the insured in Germany are used to value liabilities for annuities and pensions in Slovenia.

The DAV 1994 tables were used in the German insurance industry until 2005, when the DAV 2004 R tables were introduced (see DAV, 2005). The replacement resulted in a 10–20% increase in premiums for deferred annuities in Germany, depending on the insured's age and sex. This is a substantial increase in premium rates, and an important

question for the Slovenian insurance industry in the Solvency II framework is whether the DAV 1994 R tables are still sufficient or even appropriate for measuring the best estimate of liabilities from annuities and pensions in Slovenia. We try to answer that question here.

To achieve this goal, we implemented the Lee-Carter (LC) method and its extension, which is the current standard for actuarial modelling of future mortality. The Lee-Carter method is a powerful approach to mortality projections, as it combines a demographic model with a time-series model. In a stochastic framework, the results of LC projections consist of point and interval estimates. In this respect, the LC method allows for uncertainty in forecasts.

In this article, we apply both the LC model and the Poisson log-bilinear method, one of the latest extensions of the basic LC method, to the case of Slovenia, with mortality data for the period 1945–2007. Using a Poisson log-bilinear projection on Slovenian population data, we build selection annuity mortality tables for comparison with DAV 1994 R and DAV 2005 R. We then use the projected mortality rates for Slovenia to test the current minimum standard for valuing annuities.

The structure of this article is as follows: In Section 2 we present the main features of the stochastic LC methodology for projecting mortality. Section 3 covers data specification and calibration. In Section 4 we apply the LC and Poisson log-bilinear methods to data for Slovenia and present the results, and we also explain how kappa projections are calculated. With back-testing, we test the best fit for the projections. In Section 5 we calculate the selection effect on annuity purchasers and test the minimum standard for valuing annuity liabilities. Section 6 outlines our conclusions.

2 STOCHASTIC MORTALITY FORECASTING: THEORETICAL FRAMEWORK

2.1 The Lee-Carter model

In 1992 Lee and Carter established new standard for projecting mortality. They proposed a simple model for describing the change of mortality as a function of time index. They modelled the central death rate using the log-bilinear model (Lee & Carter, 1992)

$$\ln m_x(t) = \alpha_x + \beta_x \kappa_t + \varepsilon_{x,t} \tag{1}$$

where α_x describes the average age pattern of mortality over time, and β_x describes the deviation from the average pattern when κ_t varies. In addition, κ_t explains the evolution of the level of mortality over time t, and $\varepsilon_{x,t}$ is the error term that reflects the age-specific influences not captured by the model. It is assumed that the error term has a mean of 0 and a standard deviation of σ_ε. The basic LC assumption is that the force of mortality $\mu_x(t)$ and the central death rate $m_x(t)$ coincide, which is a direct consequence of the piecewise constant forces of assumed mortality.

The usual approach to estimating the parameters is to use the least-squares method. Furthermore, one must impose additional constraints to obtain a unique solution. The usual approach is to assume the following:

$$\sum_t \kappa_t = 0 \text{ and } \sum_x \beta_x = 1 \tag{2}$$

which forces α_x to be an average of the log of central death rates over calendar years. Once parameters α_x, β_x and κ_t are estimated (denoted by $\widehat{\alpha}_x$, $\widehat{\beta}_x$ and $\widehat{\kappa}_t$), we can forecast mortality by modelling the values of $\widehat{\kappa}_t$ in the future with a time-series approach (e.g., as a random walk with a drift or an autoregressive integrated moving average - ARIMA model).

2.2 The Poisson log-bilinear model

As several authors have noted (e.g., Brouhns et al., 2002; Sithole et al., 2000, Lee, 2000), the Lee-Carter method assumes that random errors are homoscedastic. That is, the error terms are assumed to have finite variance, and with the assumption of normality, they share the same underlying probability density function. In most cases, this assumption is violated because the logarithm of the observed mortality rate has much greater variability at older ages than at younger ages. It is therefore sensible to assume that the number of deaths follows the Poisson law with parameter (Brouhns et al., 2002):

$$D_{x,t} \sim Poisson(ETR_{x,t} \cdot \mu_x(t)) \tag{3}$$

where $D_{x,t}$ is the number of observed deaths of persons aged x in year t, $ETR_{x,t}$ is the central number of persons exposed to risk, and $\mu_x(t)$ is the force of mortality. The log of the force of mortality is $\ln \mu_x(t) = \alpha_x + \beta_x\kappa_t$, as in the LC model. The parameters have the same meaning as in the LC model.

The estimates of parameters α_x, β_x and κ_t are denoted with $\widehat{\alpha}_x$, $\widehat{\beta}_x$ and $\widehat{\kappa}_t$, obtained by maximising the log-likelihood in model , which is given by the following:

$$L(\vec{\alpha}, \vec{\beta}, \vec{\kappa}) = \sum_{x,t} D_{x,t}(\alpha_x + \beta_x\kappa_t) - ETR_{x,t} \exp(\alpha_x + \beta_x\kappa_t)) \tag{4}$$

Because of the presence of the bilinear term $\beta_x\kappa_t$, it is impossible to estimate the proposed model with commercial statistical packages that implement the Poisson regression. An option for obtaining the estimates is to use the method proposed by Goodman (1979), who suggested the iterative method for estimating log-linear models with bilinear terms. With this approach, we define the starting values for parameters as $\widehat{\alpha}_x^0 = 0$, $\widehat{\beta}_x^0 = 0$ and $\widehat{\kappa}_t^0 = 0$. The parameters are then estimated using the following iteration:

$$\hat{\alpha}_x^{n+1} = \hat{\alpha}_x^n + \frac{\sum_t (D_{x,t} - \hat{D}_{x,t}^n)}{\sum_t \hat{D}_{x,t}^n}, \quad \hat{\beta}_x^{n+1} = \hat{\beta}_x^n, \quad \hat{\kappa}_t^{n+1} = \hat{\kappa}_t^n$$

$$\hat{\kappa}_t^{n+2} = \hat{\kappa}_t^{n+1} + \frac{\sum_t (D_{x,t} - \hat{D}_{x,t}^{n+1}) \hat{\beta}_x^{n+1}}{\sum_t (\hat{\beta}_x^{n+1})^2 \hat{D}_{x,t}^n}, \hat{\beta}_x^{n+2} = \hat{\beta}_x^{n+1}, \quad \hat{\alpha}_x^{n+2} = \hat{\alpha}_x^{n+1} \tag{5}$$

$$\hat{\beta}_x^{n+3} = \hat{\beta}_x^{n+2} + \frac{\sum_t (D_{x,t} - \hat{D}_{x,t}^{n+2}) \hat{\kappa}_t^{n+2}}{\sum_t (\hat{\kappa}_t^{n+2})^2 \hat{D}_{x,t}^{n+2}}, \quad \hat{\kappa}_t^{n+3} = \hat{\kappa}_t^{n+2}, \quad \hat{\alpha}_x^{n+3} = \hat{\alpha}_x^{n+2}$$

2.3 Projecting future mortality

To obtain estimates of future mortality, one must estimate the dynamics of kappa for both men and women (Lee-Carter, 1992). As several authors have noted (e.g., Carter, 1996; Lee, 2000), κ_t can be regarded as a stochastic process, modelled by fitting an ARIMA(p,d,q) model. The dynamics of κ_t can thus be described as follows:

$$\nabla^d \kappa_t = \varphi_1 \nabla^d \kappa_t + \ldots + \varphi_p \nabla^d \kappa_t + \xi_t + \psi_1 \xi_{t-1} + \psi_q \xi_{t-q} \tag{6}$$

where $\varphi_p \neq 0$, $\psi_q \neq 0$, and ξ_t is a Gaussian white-noise process, such that $\sigma_\xi^2 > 0$. In most instances, the appropriate time-series model takes a simpler form, such as $\kappa_t = \kappa_{t-1} + c + \xi_t + \psi\xi_{t-1}$ derived from model ARIMA(0,1,1). The constant term c indicates the average annual change of κ_t and presents the forecasts of the long-term change in mortality. On the basis of the results of the time-series model, we can obtain forecasts of future mortality and its moments from the following:

$$\mu_x(t+n) = \exp(\alpha_x) \exp(\beta_x \kappa_{t+n}) \tag{7}$$

3 DATA

Population mortality data by age and sex for the period 1971–2008 were provided by the Statistical Office of the Republic of Slovenia. For the 1971–1980 period, there are some minor discrepancies between the data used and the official cumulative data for the same years, especially for 1972 and 1973. The discrepancies are in the range of 10 persons, which is negligible.

Again, we denote by $ETR_{x,t}$ the exposure to risk at age x on the last birthday during year t. The exposure to risk refers to the total number of person-years in a given population over a calendar year, and it is estimated by the number of the population aged x in the middle of the calendar year (1 July of each year); that is, those who reached age x between 1 July of the previous year and 30 June of the observing year. By $D_{x,t}$ we denote the number of

deaths recorded at age x last birthday during calendar year t. Then, the maximum likelihood estimator for $\widehat{\mu}_x(x)$ (force of mortality) equals

$$\widehat{\mu}_x(t) = \frac{D_{x,t}}{ETR_{x,t}} \tag{8}$$

Assuming a constant force of mortality for noninteger years, we have $\widehat{\mu}_x(t) = \widehat{m}_x(t)$ (see Pitacco et al., 2009). With this assumption, we can construct Slovenian mortality data for the period 1971–2008.

For our analysis, we also needed mortality data for the Slovenian population before 1971. The Human Mortality Database (HMD) contains average central mortality rates for 1930–1933, 1948–1952, 1952–1954, and 1960–1962. We used this information to interpolate $\widehat{m}_x(t)$ for the years 1945–1970. We calculated a log regression line between 1932 and 1985 and made an interpolation between 1945 and 1970 using a 95% confidence interval. We chose 1945 as a starting year for our analysis because the generation born before this year is highly likely to have already retired.

Slovenian mortality data have some irregularities that had to be adjusted before we could use them for forecasting. For example, at very old ages, the data have very low risk exposures, which leads to large sampling errors and highly volatile crude death rates. For example, risk exposures for men varied between 567 in 1971 and 1300 in 2007. For the 1971–1980 period, no data are available for age groups older than 85. Therefore, we needed a method to extrapolate a survival function at very old ages, without requiring accurate mortality data for that part of the population. Recent mortality studies suggest that the force of mortality is slowly increasing at very old ages and approaching a relatively flat shape (e.g., Pitacco et al., 2009). In other words, the exponential rate of the mortality increase at very old ages is not constant but declines.

We apply the method that Denuit and Goderniaux (2005) proposed to extrapolate death rates at very old ages. Following this approach, the death rates for very old ages were estimated according to the logistic formula proposed here. Parameters were chosen in such a way as to maximise the fit.

The log-quadratic regression model is defined as follows:

$$\ln \hat{q}_x(t) = a_t + b_t x + c_t x^2 + \varepsilon_{x,t} \tag{9}$$

where the one-year death probability at time t with $\varepsilon_{x,1}$ is independent and normally distributed, with a mean of 0 and variance of σ^2. If ω is an age limit, then we have a constraint: $q_\omega(t) = 1$. To ensure the concave behaviour of $\ln \hat{q}_x(t)$, we implemented a second constraint:

$$\frac{\partial}{\partial x} q_x(t)\Big|_{x=\omega} = 0 \tag{10}$$

We obtained the optimal fit (highest R^2) with $\omega = 130$ and a starting smoothing age $x = 130$. However, we used only the results that we obtained from age 85 onward.

Furthermore, some deaths rates were equal to 0 (i.e., there were no deaths in the observed period), which happens quite often at younger ages because of the small population. Because we used logarithms of death rates for forecasting, we implemented adjustment techniques to obtain positive values. In particular, we used interpolation techniques with neighbour central death rates to obtain the best estimate for such cases.

4 FORECASTING MORTALITY USING POPULATION MORTALITY STATISTICS FOR SLOVENIA

In this section, we present the results using the stochastic methods introduced in Section 2. On the basis of back-testing, we decided on two methods for projecting future mortality: the Lee-Carter model and the Poisson log-bilinear model based on the data from 1971 to 2008 described in Section 3. The code for this section was programmed using Matlab software.

4.1 The original Lee-Carter model vs. the Poisson log-bilinear model

First, we present the results of the model introduced in Brouhns et al. (2002) using the Poisson log-bilinear regression approach. The results of that model are compared with the results of the original Lee-Carter (1992) model. In the analysis we used data from the Slovenian Statistical Office, adjusted as described in Section 3. The results obtained from the Poisson log-bilinear model are represented in Figure 1 by a green (light) line, and the results obtained from the original Lee-Carter model are represented by a blue (dark) line.

As Figure 1 shows, the betas from both methods exhibit highly erratic behaviour, regardless of the method used. This is mainly a consequence of the small population and relatively low number of both exposures and deaths in the Slovenian population as compared with larger countries. Figure 1 shows that over the past 40 years, the biggest improvements in mortality were in the age group of minors, especially newborns and children between the ages of 10 and 14 years. In this age group, the discrepancy between the LC and Poisson log-bilinear models is also greatest. In the case of the Poisson log-bilinear model, beta is somewhat larger for this age group than in the LC method, and it is slightly lower for most of the other age groups. The betas indicate trends similar to those in other countries, with mortality improvements being greatest in the lower age groups.

Figure 1: *Beta(x) as a function of age (male): the Poisson vs. the LC model*

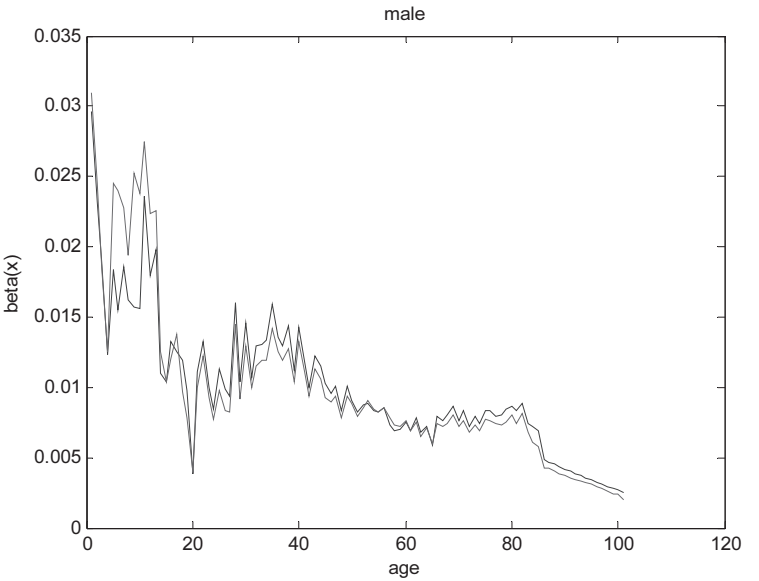
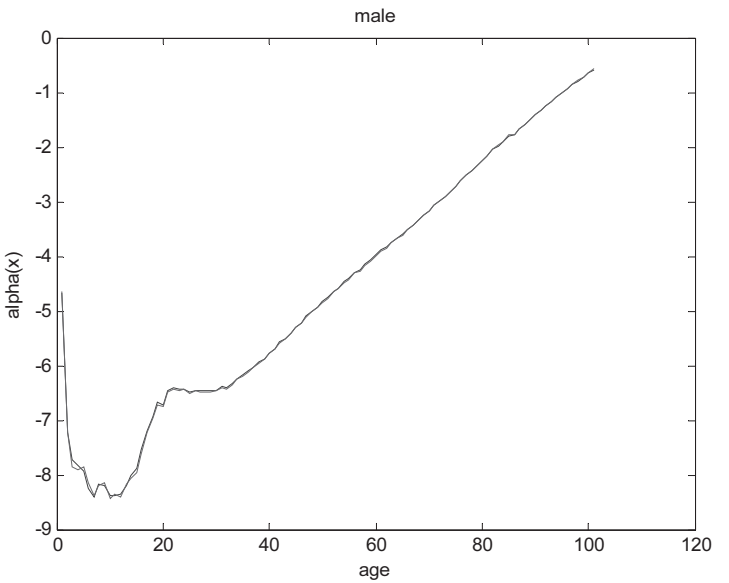


Figure 2: *Alpha as a function of age (male): the Poisson vs. the LC model*



The alphas in both models indicate that there are hardly any differences in the calculated values. The only difference is a small discrepancy in alphas for children between the ages of 2 and 10 years. Overall, the results of both models are similar to those for other

countries. Mortality is relatively high for newborns and drops considerably for minors. For male teenagers and young adults, mortality increases with a noticeable hump (i.e., the testosterone hump) around age 20. After that age, mortality is constant or slightly decreases until age 30, when it starts to increase again almost linearly into the oldest ages, as observed by Cairns et al. (2006).

Figure 3: *Kappa as a function of year (male): the Poisson vs. the LC model*

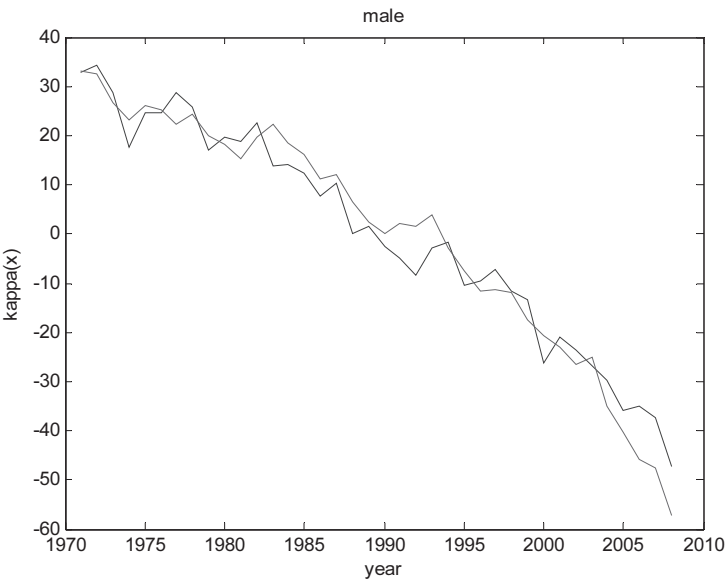


Figure 3 shows that with both methods, kappa decreases substantially over the observed period (1971–2008). During that time, there is a continuous improvement in mortality for all age groups (see also Figure 1). Given the average value of beta of 0.01, mortality has, on average, more than halved in the observed period. Of course, some age groups (e.g., minors) experienced a much greater mortality decline than the average, whereas older age groups had improvements below or well below the average. Looking at the trend for kappa, we can see that with the LC method, kappa decreases almost linearly, whereas for the Poisson log-bilinear model, it seems to increase at an even higher rate and exhibits a mild curvature.

Turning to the results for females, the trend in kappa is similar to that for males. Kappa decreases almost linearly over time under both methods. Again, both methods yield similar results, with only slight differences in calculated values for some years. Overall, we can conclude that the results of the two methods are relatively robust for the values of kappa. The results for females' alpha, beta, and kappa are presented in Appendix 1.

4.2 Projecting kappa using the Poisson log-bilinear model

In this section, we present the results of projecting kappa using the Poisson log-bilinear model (Brouhns et al., 2002). In this case, the value of *c* is equal to –2.43 (for a definition of *c*, see Section 2.3). To check the validity of the model, we examined the statistical properties of the residuals. Table 5 (in Appendix 2) shows that we cannot reject the hypothesis of normally distributed residuals. Both the Jarque-Berra test and the values of kurtosis and skewness indicate that the normal distribution is a good approximation for the distribution of the residuals. Furthermore, in looking at Q-statistics for autocorrelation, we observed that there is no statistically significant autocorrelation.

The previous analysis suggested that for Slovenian mortality statistics, a random walk with drift model is suitable for modelling the estimated κ_t . This is not surprising, as a similar observation can be found in many other countries (see, e.g., Tuljapurkar et al., 2000).

Figure 4: *Kappa(t) as a function of time (males): the Poisson model*

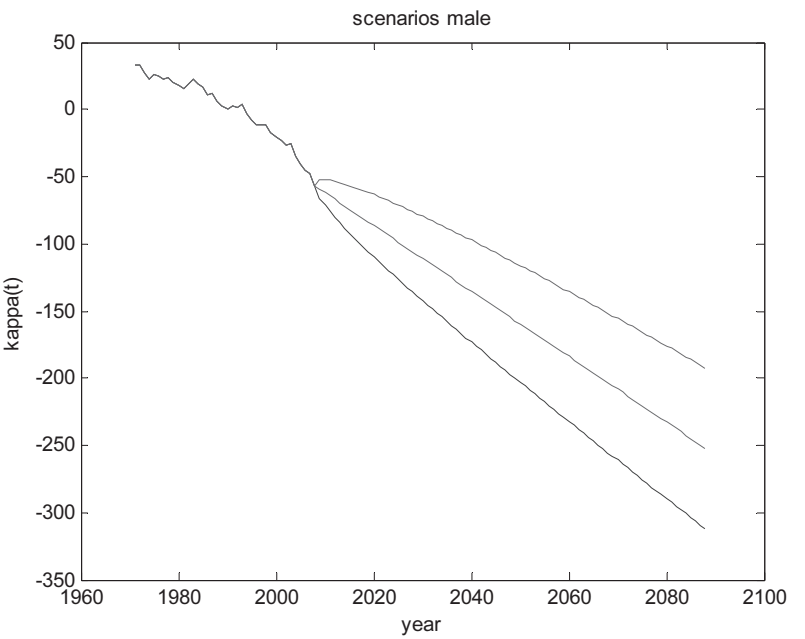
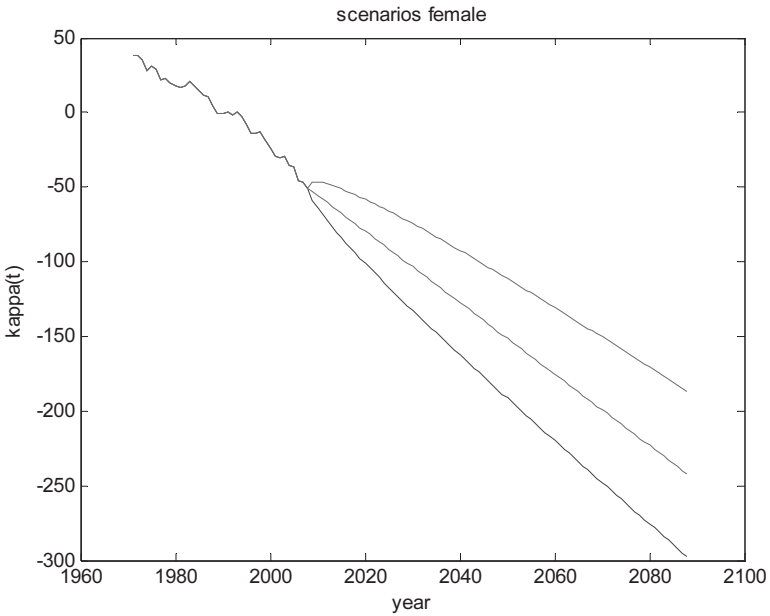


Figure 4 reveals a fairly strong downward trend in mortality. For the low-mortality scenario, the trend is somewhat higher as a result of the greater-than-expected decrease in kappa. Likewise, for the high-mortality scenario, the decrease in kappa is somewhat lower. We can see that in both cases the trend is negative, which means that the reduction in kappa is highly statistically significant.

Let us now consider the result for females. Again, the ARIMA(0,1,1) proves adequate for modelling the dynamics of kappa.

Figure 5: *Kappa(t) as a function of time (females): the Poisson model*



As one can see from the corresponding table in Appendix 2, the residuals obtained using the ARIMA(0,1,1) model are uncorrelated and approximately normally distributed. Figure 5 shows that the difference between scenarios increases in time, but the estimates remain fairly close.

4.3 Projecting kappa using the LC model

We now consider the case of modelling kappa under the original Lee-Carter model. We can see that the simple ARIMA(0,1,0) process seems to be appropriate for the estimated κ_t for males. The results, presented in Table 5 (in Appendix 2), indicate that the residuals are not autocorrelated, whereas the skewness, kurtosis, and Jarque-Berra tests indicate that the hypothesis of normally distributed residuals cannot be rejected. Another important consequence of the residual test is that, in the LC model, the standard deviation of residuals is greater than in the Poisson log-bilinear model. The difference between the forecasts is thus much smaller in the case of the Poisson log-bilinear model.

The results reveal that the trend is somewhat lower than for the Poisson model; in contrast, though, the standard deviation of the residuals is greater than in the Poisson model. The values for kappa under the high-mortality scenario are thus greater in the original LC model than in the Poisson model. The same goes for the central tendency scenario. The values for the low-mortality scenario in the LC model are comparable with the values in the Poisson model.

In contrast to males, the dynamics for females cannot be modelled as an ARIMA(0,1,0) model but can be fitted best by an ARMA(2,2) model. In this case, as presented in Table 6 (in Appendix 2), the residuals are not autocorrelated (at least up to the relevant number of lags) and can be assumed to be normally distributed.

Once again, we observe that the standard deviation of kappa estimates is significantly higher than in the Poisson log-bilinear model. This could be an argument in favour of using Brouhns et al. to model future mortality on Slovenian mortality statistics. Namely, with the Poisson log-bilinear model, the confidence interval of estimates is much narrower than in the LC model.

4.4 Back-testing

For future projections, we needed a quantitative assessment of both models before deciding which model would better capture the mortality trend. Of course, we could not do this graphically, so we used the back-testing technique to determine which model would be most appropriate for estimating future mortality. More precisely, we tested the models against real data (in our case, number of deaths) for the period 2001–2008. In the first step, we fit the model parameters to the data for the period 1971–2000. In the second step, we used the values of the parameters obtained in step 1 to predict the number of deaths $D_{x,t}$ in the period 2001–2008. We then used several standard indicators of fit to compare the methods.

Table 1: *Comparison of methods using back-testing for the 2001–2008 period (males)*

	LC	Poisson log-bilinear
MSE	21	20.8
MPE	11.7	11.6
R^2	0.955	0.96

Source: SORS

First, we examine the results for males for the period 2001–2008. A comparison of the methods using back-testing for that period (males) in Table 1 reveals the supremacy of the Poisson log-bilinear method, which is even more convincing for females. Of all variation in the number of deaths for the period 2001–2008, 99% can be explained by this method (see Table 2).

Table 2: *Comparison of methods using back-testing for the 2001–2008 period (females)*

	LC	Poisson log-bilinear
MSE	18.5	11.5
MPE	11.2	7
R^2	0.975	0.990

Source: SORS

On the basis of the back-testing analysis, we concluded that the Poisson log-bilinear model fit the actual central death rates better than the original LC model, and therefore we used that model to forecast Slovenian population mortality.

Thus, when projecting the values of kappa under different scenarios for obtaining the estimates of future mortality, we can use the following relationship:

$$m_x(2008+t) = m_x(2008)\exp(\beta_x(\kappa_{2008+t} - \kappa_{2008})) \tag{11}$$

where m_x (2008 + t) is the central death rate for year (2008 + t) and age x. Formula is essentially an extrapolation of the classical Lee-Carter model using the projections of kappa obtained from ARIMA models. In determining future mortality, we must take into account the uncertainty of our estimates. We therefore constructed three scenarios that differ with respect to kappa values used when making projections. Under the best-estimate scenario, we determined future values of kappa by taking kappa to be equal to the expected value derived from the ARIMA model. Using equation , we yielded future values of kappa through the following relationship: $\kappa_{2008+t} = \kappa_{2008} + ct$.

In the high-mortality scenario, we obtained future values of kappa by using the following relationship: $\kappa_{2008+t} = \kappa_{2008} + ct + 2\sigma_\varepsilon\sqrt{t}$. In the low-mortality scenario, we obtained future values of kappa by assuming lower-than-expected values of kappa. In this case, we obtained the future values of kappa with $\kappa_{2008+t} = \kappa_{2008} + ct - 2\sigma_\varepsilon\sqrt{t}$.

5 TESTING THE BEST-ESTIMATE VALUATION OF A LIFE ANNUITY

5.1 Cohort vs. period life tables

To calculate the present value of a future obligation arising from life annuity payments, actuaries must develop a life table (also called a mortality table or actuarial table). A life table shows for each age x the probability that a person of that age will die before his or her next birthday (denoted as q_x). Life tables are derived from observed and projected mortality rates, which can be presented in the following matrix:

$$\begin{pmatrix} q_{x_{\min}}(t_0)\cdots & q_{x_{\min}}(t_n) & \cdots & q_{x_{\min}}(t_{\max}) \\ \vdots & & & \\ \vdots & & & \\ q_{x_{\max}}(t_0) & q_{x_{\max}}(t_n) & & q_{x_{\max}}(t_{\max}) \end{pmatrix} \tag{12}$$

where
$\{q_x(t)\}$, $t \in (t_0,...,t_n)$ represents observed mortality rates, and $\{q_x(t)\}$, $t \in (t_n + 1,...,t_{max})$ represents projected mortality rates. By t_n we denote the base year from which projections are made. The sequence $q_x(t)$, $q_{x+1}(t + 1)$,... is a cohort table. The sequence $q_x(t)$, $q_{x+1}(t)$, $q_{x+2}(t)$... is a period table. This leads to the construction of two types of life tables.

In a period life table, the present value of a life annuity takes into account age-specific mortality rates at age x and older ages observed in a given calendar year. So a period life table includes different generations in a single table. In a cohort life table, a life annuity is calculated on the basis of observed birth cohort, which means that for each generation, we can construct one life table. Cohort life tables better explain improvements in mortality for each generation separately, so for best-estimate calculations, cohort life tables are used (see Pitacco et al., 2009).

To calculate the age cohort life table, we first chose the base cohort birth year τ. We then calculated the life table by taking diagonal probabilities from birth year τ as follows:

$$l_{x+1}(\tau) = (1 - q_x(\tau + x)) \cdot l_x(\tau) \tag{13}$$

We can calculate an annuity of size 1 that is payable yearly at the beginning of each year while an insured is alive from the following:

$$\ddot{a}_x(\tau) = \sum_{k=0}^{x-\omega} \left(\begin{cases} 1, & k = 0 \\ \prod_{j=0}^{k-1} (1 - q_{x+j}(\tau + x + j)), & k > 0 \end{cases} \right) \cdot (1+i)^{-k} \tag{14}$$

5.2 Selection effect

The standardised mortality ratio (SMR) is used as an index for comparing mortality experiences between two groups: actual deaths in a particular population (e.g., life annuity owners) with expected deaths, if "standard" age-specific mortality rates were to be applied. The SMR is defined as follows:

$$SMR = \frac{\sum ETR_{xt} \hat{m}_x(t)}{\sum ETR_{xt} \hat{m}_x^{\text{stand}}(t)} \tag{15}$$

A life annuity purchaser is, most likely, a healthy person with particularly low mortality in the first years of the life annuity payment and, in general, a longer-than-average expected lifetime. Therefore, to calculate the best estimate of an insurance annuity, we had to adjust mortality projections to include this effect in the projections. Pitacco et al. (2009) suggest the following model for age 60 and older:

$$\ln \hat{m}_x^{LIM}(t) = f(x) + \ln \hat{m}_x^{HMD}(t) + \varepsilon_{xt} \tag{16}$$

Here we denote by $\hat{m}_x^{HMD}(t)$ the population central death rates and by $\hat{m}_x^{LIM}(t)$ the annuity purchaser central death rates. This leads us to produce SMR in the form $e^{\hat{f}(x)}$, which can be used to adopt mortality projections for the insurance market. Slovenia mortality experience for annuity purchasers is not directly available, and because pension reform started only 10 years ago, there are no adequate statistical data for the conclusion regard-

ing the selection effect. As a result, we chose an alternative solution introduced by the Associazione Nazionale fra la Imprese Assicuratrici (2005), which has been used to build mortality statistics for the Italian insurance annuity industry.

The idea is to use SMR from another population with similar characteristics as the population for which we want to introduce the selection effect. As we see from, in general, *SMR* depends on age. Let us denote by $SMR_x^{RC}(t)$ the reference country's standardised mortality ratio between the insured and general population for the particular year *t*. We can then calculate life insurance market central death rates as follows:

$$\widehat{m}_x^{LIM}(t) = SMR_x^{RC}(t) \cdot \widehat{m}_x^{HMD}(t) \tag{17}$$

5.3 Applying the selection effect to Slovenian mortality projections

To obtain $SMR_x^{RC}(t)$, we included an element of selection that emerged from data from the United Kingdom, where annuity and pension market income is well developed. We used UK data for the period 1999–2002 collected by the Continuous Mortality Investigation Bureau and published in number 23 of the Continuous Mortality Investigation Reports (2009), which pertains to the experience of portfolios of immediate and deferred life annuities. The 1999–2002 mortality investigation presents the so-called 00 series base mortality tables adopted by UK actuaries. The statistical base is extensive: it involves more than 20 million lives exposed to risk.

In particular, we used the mortality investigation of life office pensioners (insured to deferred annuities) - PNM00 tables for men and PNF00 tables for women, which show the mortality rates for each age from 20 to 120 years, distinguished between lives (i.e., heads insured) and amounts (i.e., weighted by the benefit).

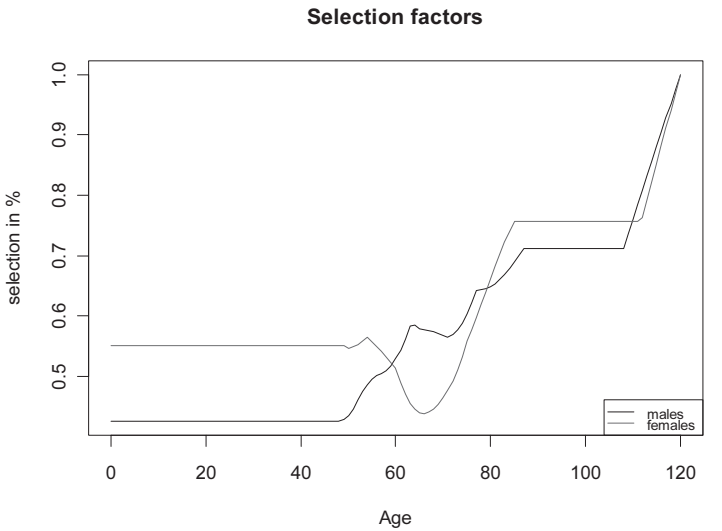
By comparing the mortality of UK insured lives with those of the total UK population (taken from English Life Table No. 16, 2000–2002), we were able to quantify the increased survival of the insured population. To take into account the impact of economic wealth of the insured on selection, we weighted mortality rates by the size of annuity, as has been statistically proved in other markets (e.g., Germany; see DAV, 2005).

Such a selection factor is structured to represent the mortality of the insured's deferred annuity. In the case of an immediate annuity, further selectivity should be added. As a result, we added an extra selection factor, calculated as the ratio between the mortality of deferred annuity owners and immediate annuitants in the United Kingdom. Figure 6 shows the combined selection factors for immediate annuitants used for the population mortality tables for Slovenia.

Starting from the particular generation cohort life table derived from historical data and stochastic projections described in the previous section, and then in applying the selection factors, we obtained the projected and selected mortality table for this particular

generation. Such derived life tables are considered the best estimate of an annuity purchaser in Slovenia. These rates can be compared with the current minimum standard: DAV 1994 R.

Figure 6: *Selection factor for Slovenia annuity owners*



5.4 Testing the minimum standard

In this section, we compare the Slovenian annuity life table with DAV 2004 and DAV 1994. Looking at the results presented in Table 3, the net single premium for an annuity based on the Poisson model is up to 12% less than the DAV 2004 single premium annuities. This gap is a consequence of the 15% margin incorporated in the new German tables (DAV, 2005) and the fact that mortality rates derived from the Poisson model represent the best estimate of future annuitant mortality. Those tables cannot be directly compared in this respect. Under the best-estimate scenario, we determined future kappa values by taking kappa to be equal to the expected value. By comparing the low-mortality scenario with DAV 2004, we observed only minor differences in rates (a 1–7% higher single premium in the case of DAV 2004).

The comparisons also show that for current generations who have not yet retired, the DAV 1994 tables underestimate the best-estimate annuity in Slovenia by 2%. For males who will retire in the future (deferred annuitants), the difference is almost 4%. As a consequence, we believe that the DAV 1994 tables should not be used for the best-estimate valuation of annuity liabilities in the Solvency II framework.

Table 3: *Immediate annuity: Age at issue 60/birth year 1950 (annuity starts in 2010)*

Net single premium	R94	R04	Poisson model, central rates	Poisson, low-mortality scenario	R94/Poisson
Male	18.10488	20.36943	18.50437	18.98097	0.978
Female	20.49378	22.00024	20.98405	21.76160	0.977

Table 4: *Deferred annuity: Age at issue 60/birth year 1980 (annuity starts in 2040)*

Net single premium	R94	R04	Poisson model, central rates	Poisson, low-mortality scenario	R94/Poisson
Male	19.72056	22.94550	20.46949	21.28829	0.963
Female	22.81581	24.42694	22.87369	23.58179	0.997

Notes: R94 – DAV 1994 annuity life table; R04 – DAV 2004 annuity life table; Poisson model – Slovenian annuity life table based on the Poisson log-bilinear model.

6 CONCLUSION

In this article, we have presented an application of the Lee-Carter methodology to calculate the best-estimate value of an insurance annuity in Slovenia. In particular, we focused on forecasting life expectancies on a time-series basis. We tested two different stochastic methods for forecasting mortality (basic Lee-Carter and Poisson log-bilinear). On the basis of back-testing analysis, we concluded that the Poisson log-bilinear model provides a better fit than the original Lee-Carter model for past observed central death rates for Slovenia. We therefore used a Poisson log-bilinear model to forecast mortality. Given that a life annuity purchaser is, most likely, a healthy person with longer-than-average life expectancy, we also incorporated the selection effect into the results. Because Slovenian mortality statistics for annuity purchasers are not directly available, we chose selection statistics from the UK experience and compared them with German statistics. By multiplying the selection factor, which depends on age, with cohort population mortality rates, we derived the best estimate of selected mortality rates for an annuity purchaser. We then compared those rates with the current minimum standard in Slovenia: the DAV 1994 R mortality rates.

The net single premium based on the Poisson model is 2–4% higher than that calculated by the current minimum standard in Slovenia. Therefore, the DAV 1994 R annuity tables are inappropriate for the best-estimate valuation of annuity liabilities in the Solvency II framework. In other words, technical provisions for annuities based on the DAV 1994 R tables are underestimated by 2–4%, which is not insignificant.

After 21 December 2012, the use of only unisex tables will be allowed for premium calculation. This is also the case for life annuities. In this respect, further research is needed to take into account male and female selection in the Poisson framework.

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APPENDIX 1: Comparing the results of both methods for females' alphas and betas

Figure 7: *Beta(x) as a function of age (females): the Poisson vs. the LC model*

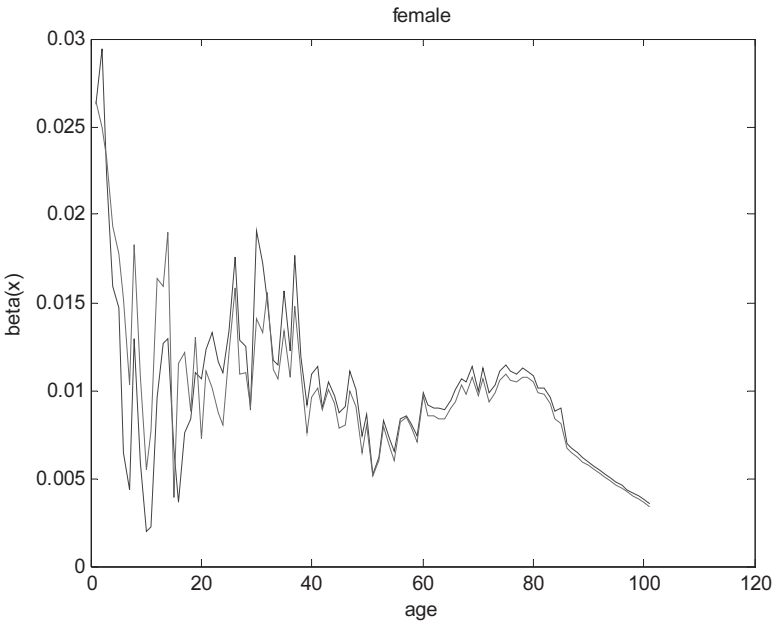


Figure 8: *Alpha as a function of age (females): the Poisson vs. the LC model*

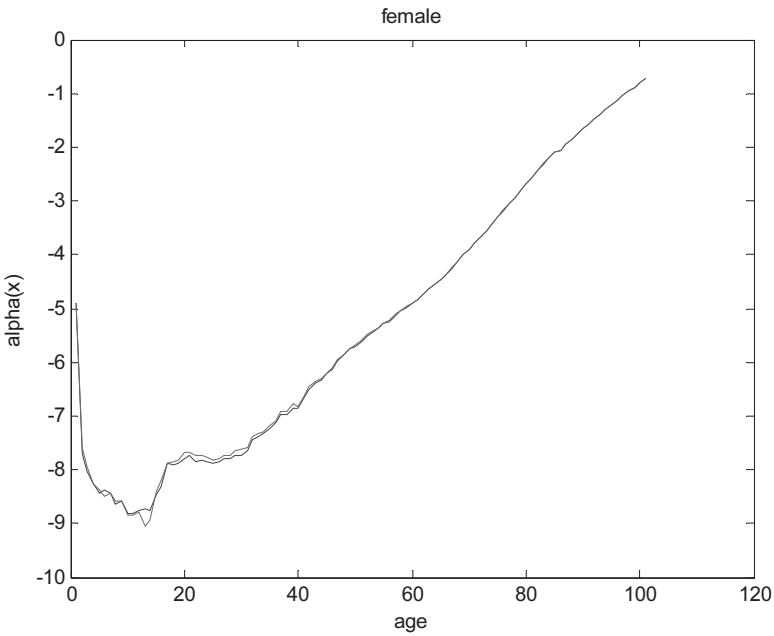
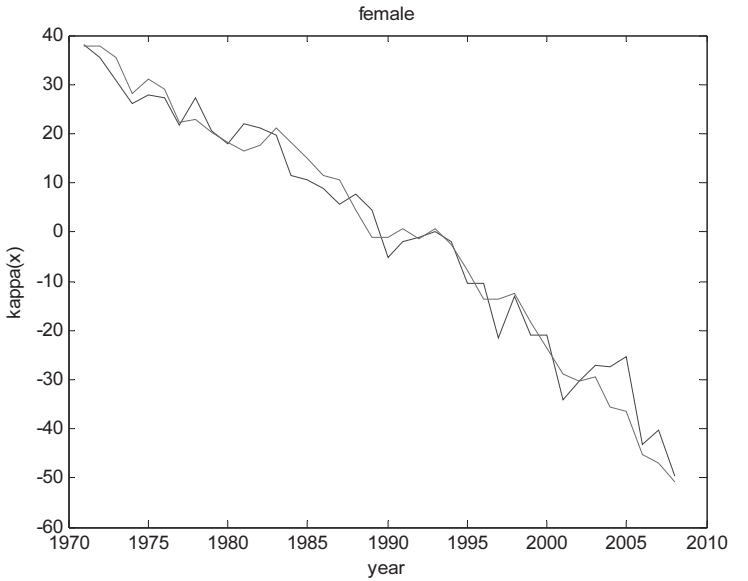


Figure 9: *Kappa as a function of year (females): the Poisson vs. the LC model*



APPENDIX 2: Residuals

Table 5: *Summary statistics for the ARIMA model*

	Poisson model		LC model	
	Males	**Females**	**Males**	**Females**
Mean	2.88E-16	-3.24E-16	-5.96E-16	-0.052288
Median	-0.3075432	0.199426	-3.33E-15	0.183130
Maximum	6.586123	6.081238	10.12461	7.569289
Minimum	-7.556903	-6.402445	-11.78582	-11.08728
Std. Dev.	3.349367	3.093559	4.889802	4.813961
Skewness	-0.049737	-0.028232	-0.409455	-0.559959
Kurtosis	2.642194	2.224110	3.041657	2.718712
Jarque-Bera	0.212627	0.933006	1.036536	2.000007
Probability	0.899143	0.627192	0.595551	0.367878
Sum	1.95E-14	-7.99E-15	-2.13E-14	-1.882381
Sum Sq. Dev.	403.8574	344.5238	860.7658	811.0976
No. of observations	37	37	37	36

Source: SORS

Table 6: *LC model residuals for kappa*

	Males				Females			
	AC	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**
1	-0.210	-0.210	1.7712	0.183	0.105	0.105	0.4340	0.510
2	-0.089	-0.139	2.0984	0.350	0.063	0.053	0.5952	0.743
3	0.052	0.001	2.2112	0.530	-0.331	-0.348	5.1487	0.161
4	-0.075	-0.080	2.4579	0.652	-0.093	-0.027	5.5211	0.238
5	0.304	0.295	6.6169	0.251	0.191	0.295	7.1360	0.211
6	-0.122	-0.009	7.3086	0.293	0.098	-0.073	7.5702	0.271
7	0.015	0.071	7.3194	0.396	0.108	-0.006	8.1237	0.322
8	-0.213	-0.284	9.5842	0.295	-0.359	-0.272	14.420	0.071
9	0.034	-0.025	9.6439	0.380	-0.043	0.094	14.513	0.105
10	0.201	0.054	11.807	0.298	0.108	0.266	15.128	0.127
11	-0.214	-0.094	14.338	0.215	0.408	0.219	24.232	0.012
12	0.179	0.159	16.178	0.183	0.055	-0.234	24.407	0.018
13	-0.193	-0.073	18.426	0.142	-0.038	0.097	24.493	0.027
14	0.001	-0.016	18.426	0.188	-0.227	0.035	27.701	0.016
15	-0.010	-0.211	18.433	0.241	-0.134	-0.122	28.874	0.017
16	0.070	0.151	18.770	0.281				

Table 7: *Brouhns et al. residuals for modelling kappa*

	Males				Females			
	AC	**PAC**	**Q-Stat**	**Prob**	**AC**	**PAC**	**Q-Stat**	**Prob**
1	-0.255	-0.255	2.6156	0.106	-0.072	-0.072	0.2090	0.648
2	-0.118	-0.196	3.1852	0.203	-0.161	-0.167	1.2718	0.529
3	-0.089	-0.195	3.5248	0.318	-0.050	-0.079	1.3799	0.710
4	-0.029	-0.161	3.5618	0.469	-0.013	-0.053	1.3870	0.846
5	0.261	0.181	6.6234	0.250	0.020	-0.008	1.4055	0.924
6	-0.225	-0.153	8.9865	0.174	-0.094	-0.113	1.8171	0.936
7	-0.003	-0.060	8.9868	0.254	0.130	0.114	2.6304	0.917
8	-0.177	-0.257	10.540	0.229	0.035	0.024	2.6912	0.952
9	0.160	-0.003	11.866	0.221	-0.154	-0.127	3.9196	0.917
10	0.048	-0.076	11.990	0.286	0.128	0.134	4.7928	0.905
11	-0.230	-0.238	14.929	0.186	0.155	0.160	6.1273	0.865
12	0.310	0.213	20.473	0.059	0.152	0.214	7.4576	0.826
13	-0.156	-0.042	21.929	0.056	-0.251	-0.155	11.245	0.590
14	0.046	-0.084	22.063	0.077	-0.087	-0.047	11.717	0.629
15	-0.025	-0.028	22.104	0.105	0.133	0.075	12.877	0.612
16	-0.038	-0.016	22.201	0.137	-0.064	-0.038	13.157	0.661